ECE801
Monitoring and Estimation

Introduction

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Outline

- Topics of the course
- Motivation
- Modeling Overview

Motivation - Monitoring of Critical Infrastructures

- Intelligent Transportation Systems
  - Traffic monitoring
  - Autonomous vehicles
- Power Systems
- Water networks
- Telecommunication networks

Transportation (Autonomous Vehicles)

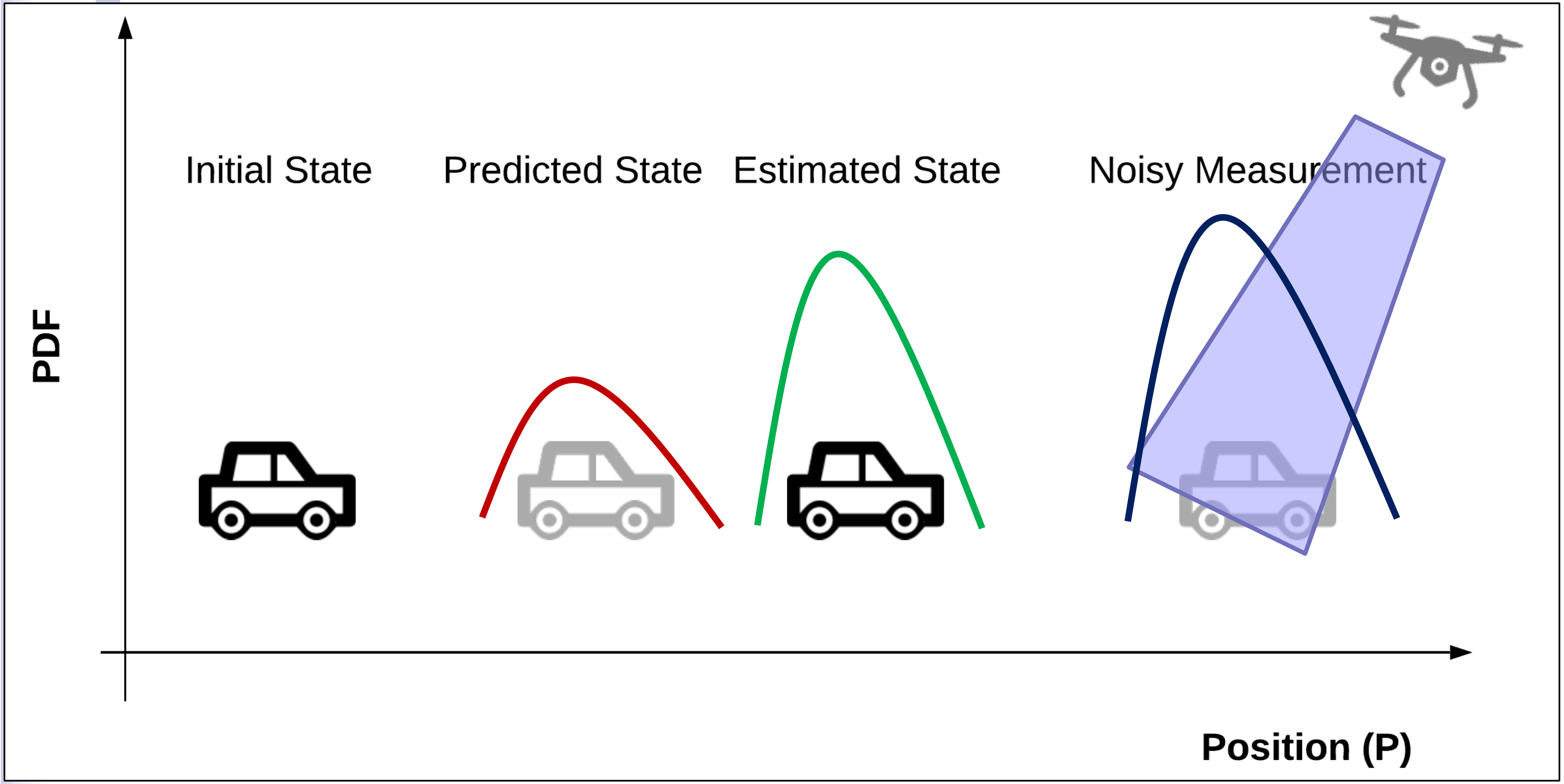
$$\dot{x} = v = \frac{dx}{dt}$$

State Update Equation

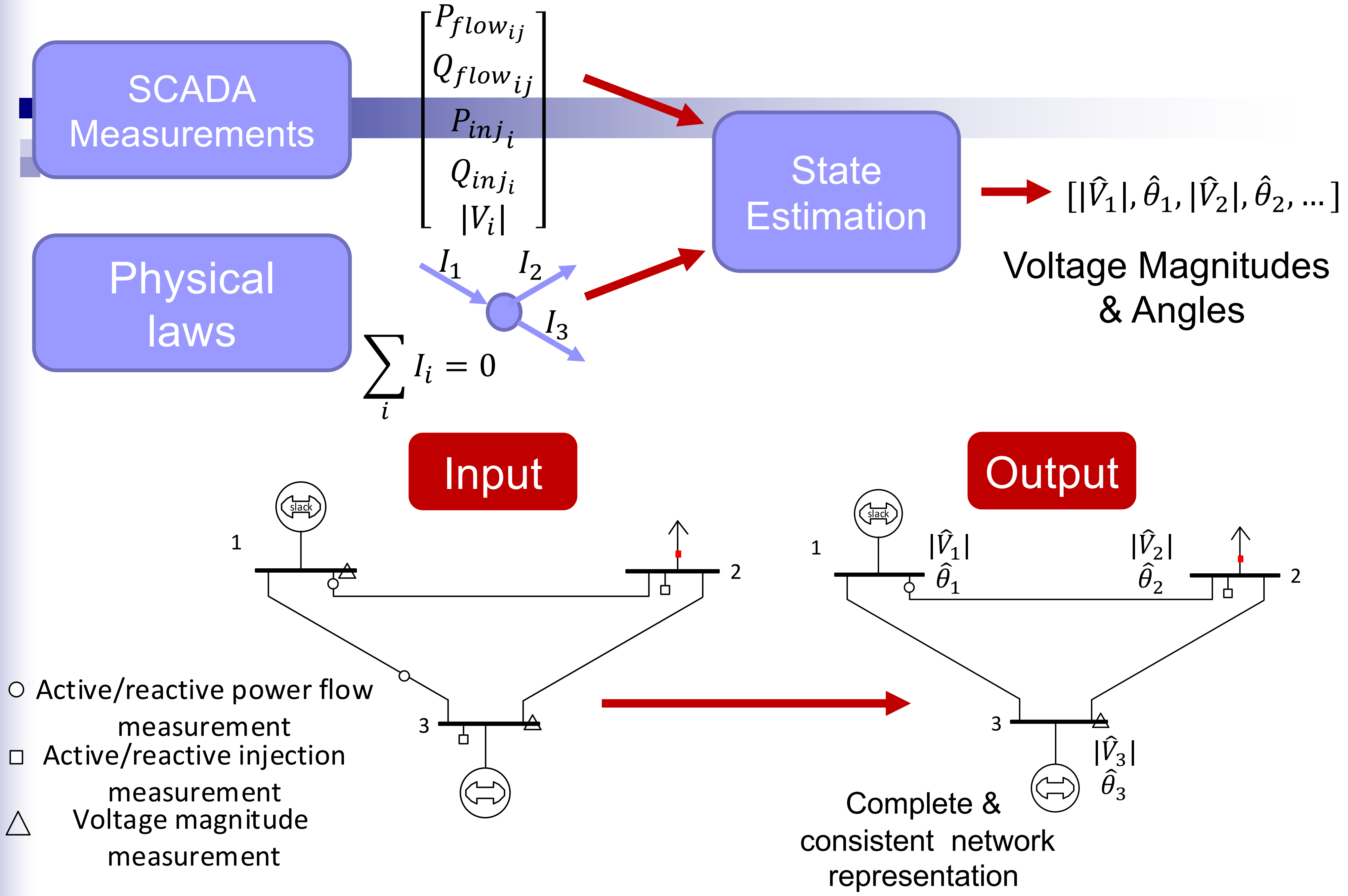
$$x_{i+1} = x_i + \Delta t x_i$$
$$\dot{x}_{i+1} = \dot{x}_i$$

State Extrapolation Equation

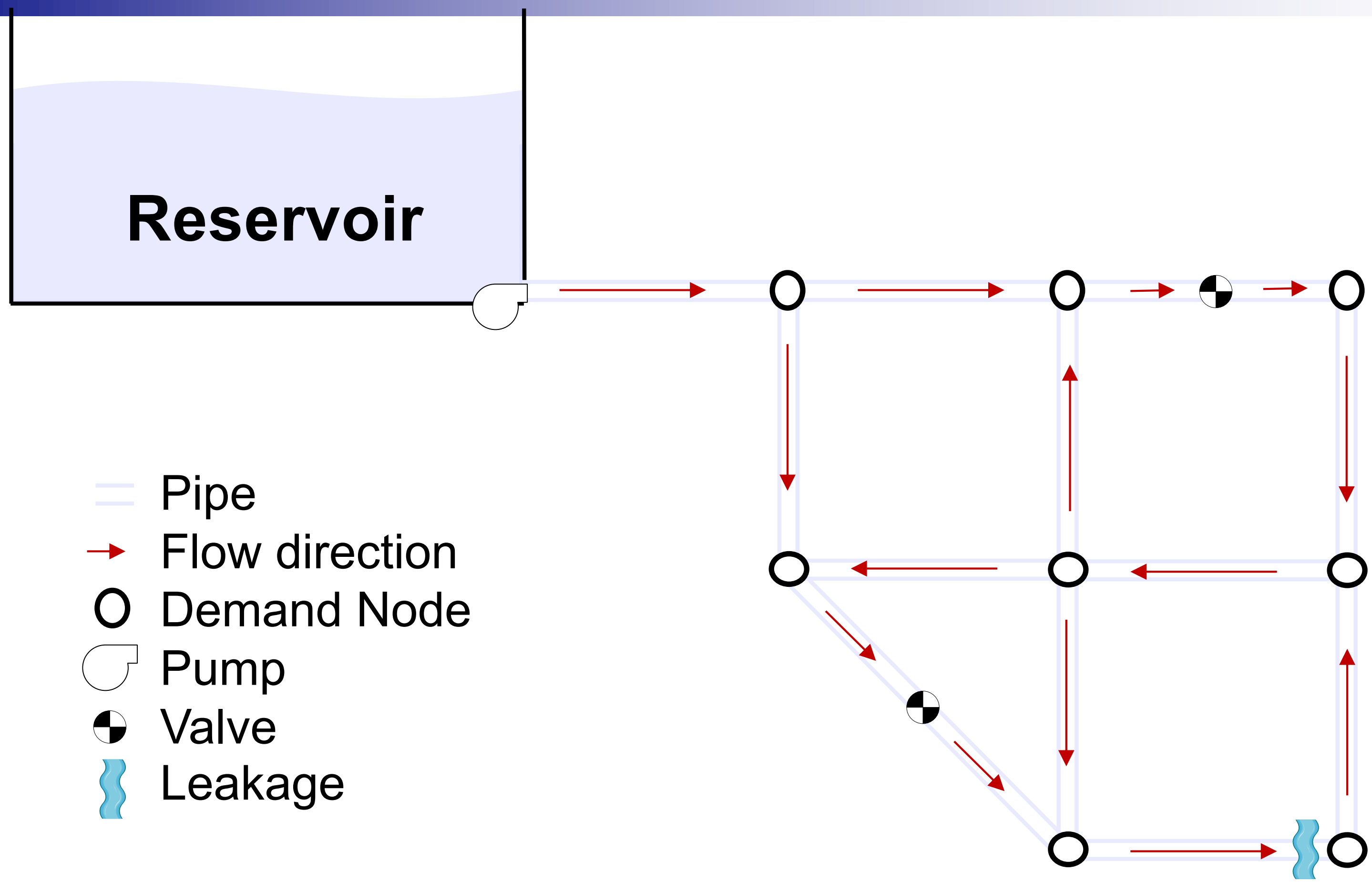
Transportation (Autonomous Vehicles)



Power Networks



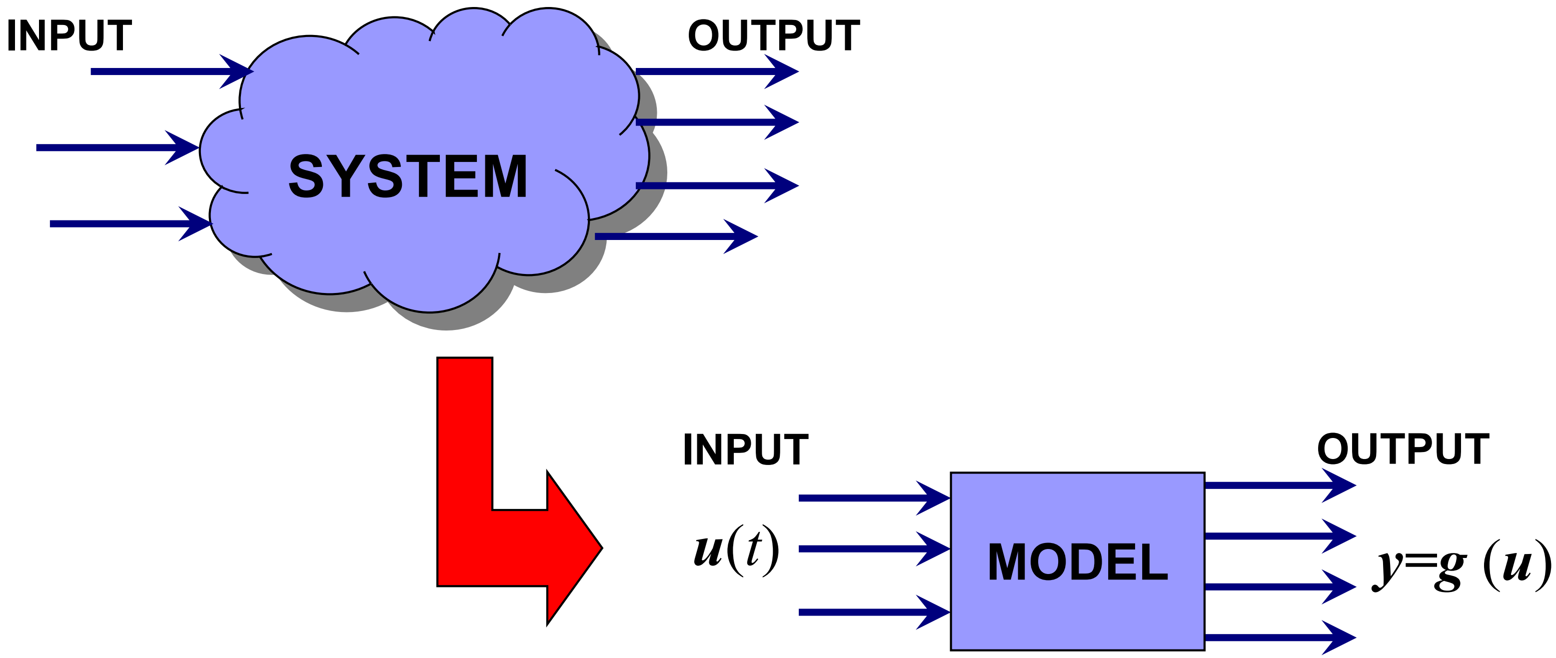
Water Networks



Modeling

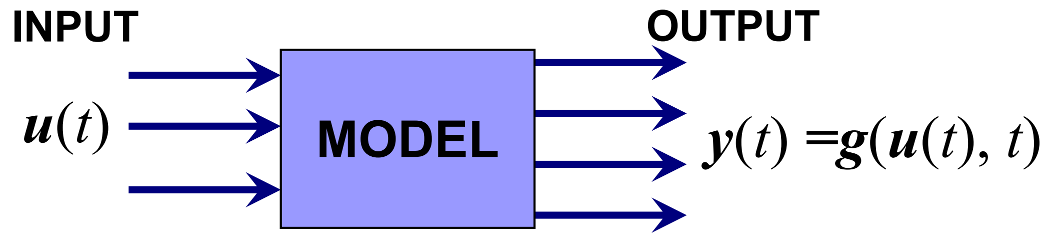
- Model
  - It is a set of equations or a piece of software (simulator) that imitates the behavior of the real system.
  - There may be several models that can capture the behavior of a system.
- Modeling is mostly an *art* and not an exact science.
  - Depending on the answers we are looking for, models can be very detailed and complex or they can be very simple.

Modeling Process



- A model predicts what the system's output would be given an input $u(t)$.
- A model is as good as its input: garbage in, garbage out!

Concept of State



- Suppose that at a time instant t_1, $u(t_1)$=a and $y(t_1)=Y$. Then, at time t_2, $u(t_2)$=a then what is $y(t_2)$=?.
- **Example:** Let

$$x(t)=x(t-1)+u(t)$$

$$y(u(t))=u(t)+x(t)+5$$

- The **state** of a system at time t_0 is the information required at t_0 such that the output $y(t)$, for all $t \geq t_0$, is uniquely determined from this information and from the input $u(t)$, $t \geq t_0$.

State Space Modeling

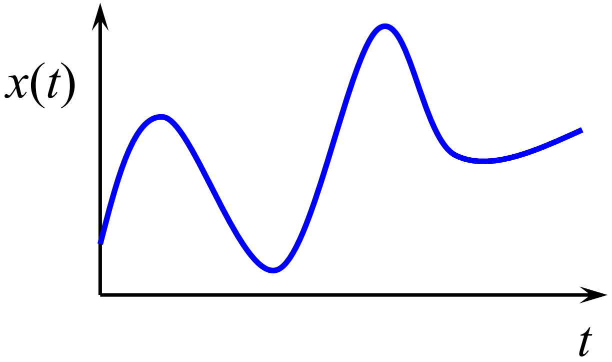
- **State equations**: The set of equations required to specify the state $x(t)$ for all $t \geq t_0$ given $x(t_0)$ and the function $u(t)$.
- **State Space X**: The set of all possible values that the state can take.
- **Examples**:

$$\dot{x}(t) = f\left(x(t), u(t), t\right) \qquad x_{k+1} = f\left(x_k, \ldots, x_0, u_k, \ldots, u_0, k\right)$$

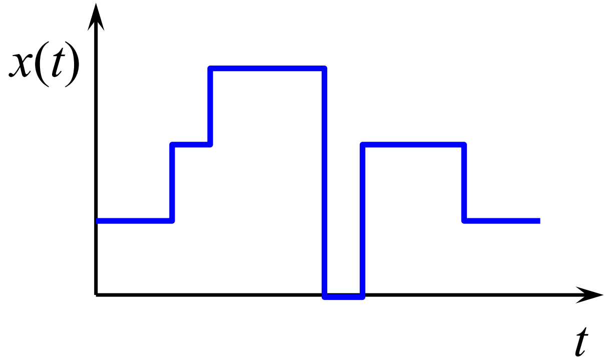
$$y(t) = g\left(x(t), u(t), t\right) \qquad y_k = g\left(x_k, \ldots, x_0, u_k, \ldots, u_0, k\right)$$

$$x(t_0) = x_0$$

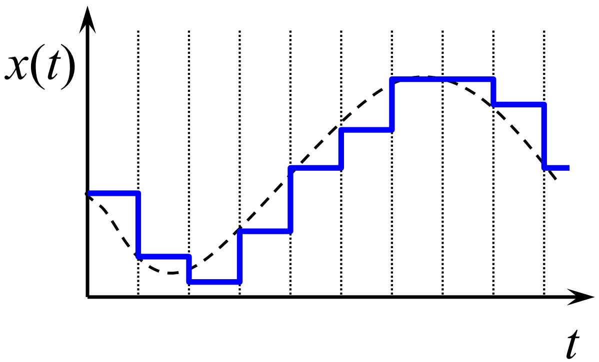
System Classification



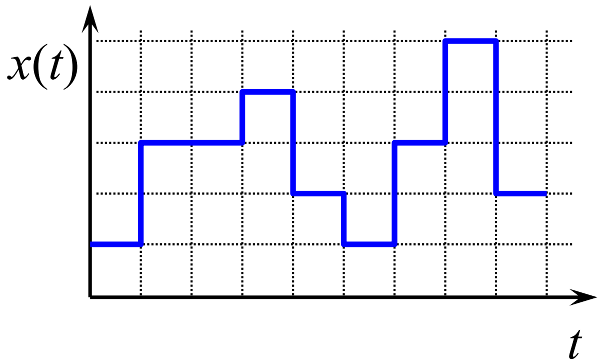
Continuous State – Continuous Time



Discrete State – Continuous Time



Continuous State – Discrete Time



Discrete State – Discrete Time

Deterministic and Stochastic Systems

- In many occasions the input functions $u(t)$ are not known exactly but we can only characterize them through some probability distribution.
  - Signal noise at a mobile receiver
  - Arrival time of customers at a bank
  - …
- If the input function is not known exactly, then the state cannot be determined exactly, but it constitutes a random variable
- A system is stochastic if at least one of its output variables is a random variable. Otherwise the system is deterministic.
- In general, the state of a stochastic system defines a random process.

Parameter vs State Estimation

- Parameter estimation refers to the estimation of the value of a constant parameter of our model (e.g. the resistance of a resistor)
- State estimation refers to estimating the state of a variables the changes over time.

Parameter estimation example

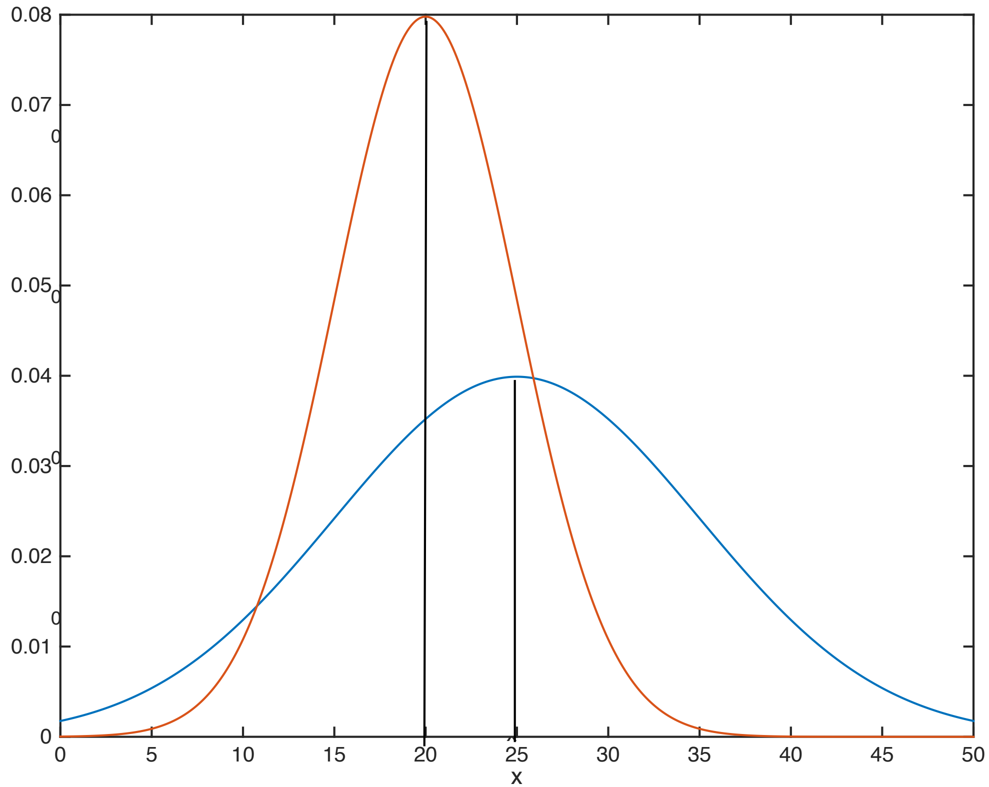
- The true value of a resistor's resistance is not known exactly. Thus we measure it with two multimeters. The obtained measurements are
  - $y_1 = x + v_{1,}$ and
  - $y_2 = x + v_2$

Where x is the resistance we are looking for and $v_1$and v_2 are the noise uncertainty of the instrument (assumed additive). Both are assumed Gaussian with 0 mean and variance σ_1^2 and σ_2^2 respectively.

Question: what is your best estimate for the value of x?

Parameter estimation example

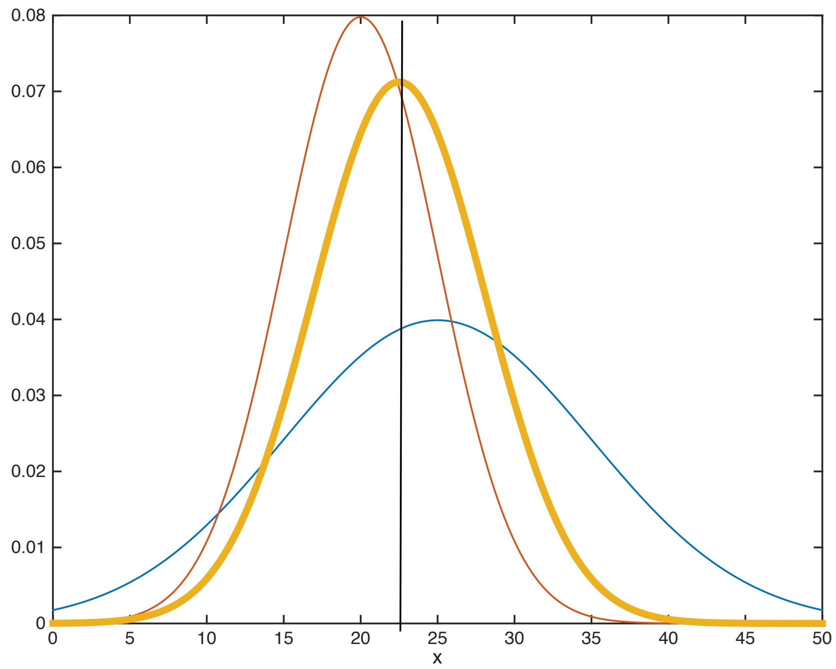
- If $y_1 = 25$ and $\sigma_1 = 10$, then $\hat{x} = y_1 = 25$
- If $y_2 = 20$ and $\sigma_2 = 5$, then maybe $\hat{x} = y_2 = 20$
- Can we do better?



Parameter estimation example

- Average: $\hat{x} = \frac{y_1 + y_2}{2} = 22.5$
- What would be the variance in this case?

$$\sigma^2 = \frac{\sigma_1^2 + \sigma_2^2}{4} = \frac{100 + 25}{4} = 31.25 \qquad \Longrightarrow \sigma = \sqrt{31.25} = 5.6$$



Parameter estimation example

- Weighted Average: $\hat{x} = w_1 y_1 + w_2 y_2$
- How can we find the weights w_1 and w_2?
- We would like to have $E[\hat{x} - x] = 0$

$$E[\hat{x} - x] = E[w_1 y_1 + w_2 y_2 - x] =$$
$$= E[w_1(x + v_1) + w_2(x + v_2) - x] = 0$$

$$\Longrightarrow w_1 x + w_2 x - x = 0 \qquad \boxed{\Longrightarrow w_1 = 1 - w_2}$$

- So, lets assume $w_1 = w$ and $w_2 = 1 - w$
- We need to find w that minimizes the estimator variance

$$J = E[(\hat{x} - x)^2] = E[(w y_1 + (1 - w)\, y_2 - x)^2]$$

Parameter estimation example

- So, what is w?

$$J = E[(\hat{x} - x)^2] = E[(wy_1 + (1 - w)\, y_2 - x)^2]$$

$$= E[(w(x + v_1) + (1 - w)\, (x + v_2) - x)^2]$$

$$= E[(wv_1 + (1 - w)\, v_2)^2]$$

$$= w^2\sigma_1^2 + (1 - w)^2\, \sigma_2^2$$

- What w minimizes J?

$$\frac{dJ}{dw} = 2w\sigma_1^2 - 2(1 - w)\, \sigma_2^2 = 0$$

- Therefore

$$w_1 = w = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}$$

$$w_2 = 1 - w = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}$$

Parameter estimation example

- Weights

$$w_1 = w = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2} = \frac{25}{125} = 0.2 \qquad w_2 = 1 - w = 0.8$$

- Weighted Average: $\hat{x} = 0.2y_1 + 0.8y_2 = 21$
- With variance $\sigma^2 = 0.2^2\sigma_1^2 + 0.8^2\sigma_2^2 = 20$, so $\sigma = \sqrt{20} = 4.5$