KOIOS Research and Innovation Center of Excellence

University of Cyprus
Department of Electrical and Computer Engineering

ECE 801 – Monitoring and Estimation

Assignment 1 (Due: 20/09/2019)

Report: Your report should be sent via email to course teaching assistant (cmenel02@ucy.ac.cy) prior the deadline and must include the usual cover page. In your report, include any comments and description you may want to add. Email subject line should only consist of "ECE801_2019". Naming format for the zip/rar file: lastName.zip/rar.

1. **[20%]**
A hypothesis-testing problem using chi-squared distributed random variables has the pdfs
$$p(y|H_0) = \begin{cases} \frac{1}{2}e^{-y/2}, y \geq 0 \\ 0\,, otherwise \end{cases}$$
for a chi-squared random variable with two degrees of freedom and
$$p(y|H_1) = \begin{cases} \frac{1}{4}ye^{-y/2}, y \geq 0 \\ 0\,, otherwise \end{cases}$$
for a chi-squared random variable with four degrees of freedom. For equally likely a prior probability, find analytically:
(a) The likelihood ration $L(y)$.
(b) P_{11}, P_{00}, P_{10} and P_{01}.
(c) The cdf of hypothesis H_0 and H_1
(d) Using MATLAB estimate P_{11}, P_{00}, P_{10} and P_{01}. Compare the estimated results with theoretically obtain answers considering different number of samples (e.g., 100, 1000, 10000 and 100000). Hint: Values of the random variable y for a chi-squared random variable with two degrees of freedom can be generated from:
$$y = -2\,ln(u)$$
where u is uniformly distributed in the interval (0,1) while for a chi-squared random variable with four degrees of freedom can be generated from:
$$y = \sum_{i=1}^{4} x_i^2$$
where each x_i is an independent, zero mean Gaussian random variable with unit variance.

2. **[20%] (Exercise 4.11 from textbook)**
Consider two hypotheses $H_0$and H_1 with the associated probability density functions
$$p_0(y) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{y^2}{2}\right)$$
and

$$p_1(y) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(y-1)^2}{2}\right)$$

a priori probabilities $\pi_0 = 0.3$, $\pi_1 = 0.7$, and costs $C_{00} = C_{11} = 0, C_{01} = 2$ and $C_{10} = 1$. Find analytically:

(a) The Bayes threshold τ_B,

(b) The MAP threshold τ_{MAP},

(c) The Neyman-Pearson threshold τ_{NP}.

Use MATLAB to generate a sequence of 1000 samples of a zero-mean, unit variance Gaussian variable. Compare each of the samples with the all the thresholds found above and maintain a running record of P_{10} for each criterion. Compare the MATLAB results with theory and explain any discrepancies.

3. [20%]

In a running sum the sample mean, and sample variance are defined for the current sample i as

$$\bar{y} = \frac{1}{i}\sum_{t=1}^{i} y_t$$

and

$$\bar{s}^2 = \frac{1}{i}\sum_{t=1}^{i} (y_t - \bar{y})^2 .$$

(a) Using MATLAB generate (i) 1000 and (ii) 10000 samples of a zero-mean, unit-variance normal random variables and plot the result for the sample mean and sample variance as a function of the sample number i. Discuss your findings.

(b) Substitute the theoretical mean for the sample mean in sample variance and plot the result together with the above findings and discuss your observations.

4. [20%] (Exercise 10.13 from textbook)

Find the ML and MAP estimates of the parameter θ, given the observation y, if

$$y = 2\theta + \theta^2 + n$$

where

$$p(n) = \begin{cases} e^{-n}, & n > 0 \\ 0, & otherwise \end{cases}$$

and

$$p(\theta) = \frac{1}{5}\delta(\theta) + \frac{4}{5}\delta(\theta - 2)$$

where δ denotes the Dirac delta function.