ECE 801 – Monitoring and Estimation

Assignment 11(Due: 17/10/2019)

Report: Your report should be sent via email to course teaching assistant (cmenel02@ucy.ac.cy) prior the deadline and must include the usual cover page. In your report, include any comments and description you may want to add. Email subject line should only consist of "ECE801_2019". Naming format for the zip/rar file: lastName.zip/rar.

1. **[25%]**
   Assume a linear dynamical system
   $$x_{k+1} = Ax_k + Bu_k$$
   $$y_k = Cx_k$$
   where $A = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.4 & 0 \\ 0.3 & 0.2 & 0.8 \end{bmatrix}$, $B = 10^{-2}\begin{bmatrix} 3 \\ 8 \\ 2 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & .5 & 0 \end{bmatrix}$

   (a) What are the eigenvalues of the system?
   (b) How many sensors does the system have and is it observable?
   (c) Assume that at some point all existing sensors have failed and you only have money to buy one sensor. Which sensor would you buy?
   (d) Given the sensor that you bought in part (c) above, design an observer such that the eigenvalues of the "closed loop" observer $A - LC$ are placed at points 0.5 and $\pm 0.02i$.

2. **[25%]**
   Assume that you run an experiment and you know that the model is given by $y = g(x) = \theta_2 x^2 + \theta_1 x + \theta_0 + w$ where w is a zero mean white noise.
   (a) Design an estimator for the parameters θ_0, θ_1, and θ_2
   (b) Design a sequential estimator for the same problem
   (c) How would your estimator change if you knew that the noise samples are Gaussian with variance σ^2 and the parameters θ_0, θ_1, and θ_2 are also Gaussian $\mathcal{N}(\theta, \Sigma_\theta)$ where $\theta \in \mathfrak{R}^3$ and $\Sigma_\theta \in \mathfrak{R}^{3\times3}$.
   (d) Design a sequential estimator for the problem in (c).

3. **[25%]**
   Let a system with 2 inputs and 2 outputs, where $y_1 = x_1 + x_3 + n_1$ and $y_2 = x_1 + x_2 + n_2$. Assume that the inputs are Gaussian with:

   $$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \sim \mathcal{N}\left(\mu_X = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \Sigma_X = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix} \right)$$

and the outputs are corrupted by Gaussian noise n as follows:

$$W = \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \sim N\left(\mu_n = \begin{bmatrix} n \\ n \end{bmatrix}, R_n = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\right)$$

Determine the ML and MAP estimators.

4. **[25%]**
   Assume that y is the time between two consecutive vehicle arrivals at the entrance of a bridge. Assume that there are two hypotheses H_0 and H_1 where H_0 is the "normal" traffic and H_1 is the "high" traffic. The prior probabilities of the two hypotheses are π_0 and π_1, respectively. The distribution of y given each of the two hypotheses is given by

$$f(y|H_0) = \begin{cases} \lambda_0 e^{-\lambda_0 y} & y \geq 0 \\ 0 & y < 0 \end{cases} \quad \text{and} \quad f(y|H_1) = \begin{cases} \lambda_1 e^{-\lambda_1 y} & y \geq 0 \\ 0 & y < 0 \end{cases}$$

$\lambda_1 > \lambda_0$.

1. Determine the MAP threshold and compute the probability of **correctly** classifying the traffic state, the probability of predicting high traffic when normal traffic is present (**false alarm**) and the probability of not detecting the high traffic (**miss event**) when $\pi_0 = 0.5$ and $\pi_1 = 0.5$, $\lambda_0 = 1$ and $\lambda_1 = 3$.
2. Determine the Bayes' threshold and compute the probability of **correctly** classifying the traffic state, the probability of predicting high traffic when normal traffic is present (**false alarm**) and the probability of not detecting the high traffic (**miss event**) when $\pi_0 = 0.6$ and $\pi_1 = 0.4$, $\lambda_0 = 1$ and $\lambda_1 = 3$ and the cost of misclassification is given by $C_f = 1$ (false alarm) and $C_m = 2$ (miss event). Compute the average Bayes' risk (assume zero cost for correct classification).
3. Determine the Neyman Pearson threshold and compute the probability of **correctly** classifying the traffic state, the probability of predicting high traffic when normal traffic is present (**false alarm**) and the probability of not detecting the high traffic (**miss event**) when the false alarm probability should not exceed, given that $\alpha_f = 0.2$.