



HMY 477 – Biomedical Optics

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Introduction

What is Biomedical Optics?

Biomedical Optics

- engineering optics for biomedical research Tromberg (UC-Irvine)
- application of optical science and technology to biomedical problems
 Bigio (BU)





Introduction



- What is Biophotonics?
- Photonics = the technology of generating, controlling and detecting light, whose quantum unit is the photon. Implies an analogy to electronics.
- Biophotonics = application of photonics to biomedical problems.
- For purposes of this course:
- biomedical optics = biophotonics



Why is biomedical optics important?



- Light can be used to reveal much about tissue noninvasively (without damaging the tissue).
- Light is non-ionizing radiation.
- Light can travel farther into tissue than you think.
- Optical fibers can be used to deliver and collect light, permitting access to remote sites within the body, mediated by endoscopes, catheters or needles.



Why is biomedical optics important?



- New properties of tissue can be measured with light, enabling new types of diagnostic measurements.
 - e.g., blood oxygenation, nuclear size, etc.
- New methods of therapy can be accomplished with light, enabling new ways to cure diseases or repair problems in tissue.
 - e.g., photodynamic therapy



Minimally-invasive diagnostics and therapeutics



Medical definition of "noninvasive"



• A procedure is noninvasive if:

- no exogenous drug is administered
- no organ barrier is violated.

• →examples:

- giving the patient an aspirin is invasive
- colonoscopy is noninvasive



"Noninvasive" colonoscopy





You're going to put that WHERE ?!?





Exploiting interactions of light with matter

Wavelengths used typically: 300-900 nm



Theories on nature of light: Light as a particle vs. Light as a wave



- Only corpuscular theory of light prevalent until 1660
- Francesco Maria Grimaldi (Bologna) described diffraction in 1660



Fig. 315. Grimaldi's Observation of Diffraction





Light as a particle



Sir Isaac Newton (1642-1727)

- Embraces corpuscular theory of light because of inability to explain rectilinear propagation in terms of waves
- Demonstrates that white light is mixture of a range of independent colors
- Different colors excite ether into characteristic vibrations---sensation of red corresponds to longer ether vibration



Light as a wave



Christiaan Huygens (1629-1695)



- Huygens' principle (Traite de la Lumière, 1678):
- Every point on a primary wavefront serves as the source of secondary spherical wavelets, such that the primary wavefront at some later time is the envelope of these wavelets. Wavelets advance with speed and frequency of primary wave at each point in space



http://id.mind.net/~zona/mstm/physics/waves/propagation/huygens1.html

Light as a wave

Thomas Young (1773-1829)

 1801-1803: double slit experiment, showing interference by light from a single source passing though two thin closely spaced slits projected on a screen far away from the slits







Light as a wave

Augustine Fresnel (1788-1827)

 1818: Developed mathematical wave theory combining concepts from Huygens' wave propagation and wave interference to describe diffraction effects from slits and small apertures





Electromagnetic wave nature of light



- Michael Faraday (1791-1865)
- 1845: demonstrated electromagnetic nature of light by showing that you can change the polarization direction of light using a strong magnetic field



Electromagnetic theory



- James Clerk Maxwell (1831-1879)
- 1873: Theory for electromagnetic wave propagation
- Light is an electromagnetic disturbance in the form of waves propagated through the ether



Quantum mechanics





Max Planck



Niels Bohr



Louis de Broglie

 1900: Max Planck postulates that oscillating electric system imparts its energy to the EM field in quanta

1905: Einstein-photoelectric effect

- Light consists of individual energy quanta, photons, that interact with electrons like particle
- 1900-1930 it becomes obvious that concepts of wave and particle must merge in submicroscopic domain
- Photons, protons, electrons, neutrons have both particle and wave manifestations
 - Particle with momentum p has associated wavelength given by p=h/ $\!\lambda$
- QM treats the manner in which light is absorbed and emitted by atoms



Heisenberg



Schrödinger





Maxwell's Equations

Name	Differential form	Integral form
Gauss's law	$\nabla \cdot \mathbf{E} = \frac{\mu}{\varepsilon_0}$	$\oint_{\partial V} \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \frac{Q(V)}{\varepsilon_0}$
Gauss's law for magnetism	$\nabla \cdot \mathbf{B} = 0$	$\oint_{\partial V} \mathbf{B} \cdot \mathrm{d}\mathbf{A} = 0$
Maxwell–Faraday equation (Faraday's law of induction)	$ abla imes \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint_{\partial S} \mathbf{E} \cdot \mathrm{d} \mathbf{l} = -\frac{\partial \Phi_S(\mathbf{B})}{\partial t}$
Ampère's circuital law (with Maxwell's correction)	$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$	$\oint_{\partial S} \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_S + \mu_0 \varepsilon_0 \frac{\partial \Phi_S(\mathbf{E})}{\partial t}$



 In a vacuum and charge free space, these equations are:

$$\nabla \cdot \mathbf{E} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

• Taking the curl of the curl equations gives:

 $\begin{aligned} \nabla\times\nabla\times\mathbf{E} \ &=\ -\frac{\partial}{\partial t}\nabla\times\mathbf{B} = -\mu_0\varepsilon_0\frac{\partial^2\mathbf{E}}{\partial t^2} \\ \nabla\times\nabla\times\mathbf{B} \ &=\ \mu_0\varepsilon_0\frac{\partial}{\partial t}\nabla\times\mathbf{E} = -\mu_o\varepsilon_o\frac{\partial^2\mathbf{B}}{\partial t^2} \end{aligned}$

• By using the vector identity $\nabla \times (\nabla \times \mathbf{V}) = \nabla (\nabla \cdot \mathbf{V}) - \nabla^2 \mathbf{V}$ • Results in the wave equations:

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} - c_0^2 \cdot \nabla^2 \mathbf{E} = 0$$
$$\frac{\partial^2 \mathbf{B}}{\partial t^2} - c_0^2 \cdot \nabla^2 \mathbf{B} = 0$$

where

$$c_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.99792458 \times 10^8 \text{ m/s}$$

• is the speed of light in free space.



One-dimensional wave equation

 $\psi(x,t) = A\cos(kx - \omega t - \delta) = \operatorname{Re}\left\{A\exp\left[j\left(kx - \omega t - \delta\right)\right]\right\}$

- Wavelength: $\lambda = 2\pi/k$
- Frequency: $f=\omega/2\pi$
- Speed: $v = \omega k^{-1} = \lambda f$,
- Phase: $\phi(x, t) = kx \omega t \delta$
- Phase shift: $\Delta \phi = 2\pi \Delta x \lambda^{-1}$
- Plane Wave

$$\psi(\vec{r}) = A \exp[j(\vec{k} \cdot \vec{r} - \omega t - \delta)]$$

Spherical Wave

$$\psi(\vec{r}, t) = \frac{A}{r} \exp\left[j\left(kr - \omega t - \delta\right)\right]$$

Cylindrical Wave

$$\psi(\vec{r}, t) = \frac{A}{\sqrt{r}} \exp\left[j\left(kr - \omega t - \delta\right)\right]$$

Points of equal phase → wavefronts





• The Gaussian wave

$$\vec{E}(r) = \vec{u}A_0\left(\frac{W_0}{W(z)}\right) \exp\left(-\frac{r^2}{W^2(z)}\right) \exp\left(-jkz - jk\frac{r^2}{2R(z)} + j\zeta(z)\right)$$

- Radius at which E drops by 1/e and I by 1/e²:
- Radius at z=0:
- Radius of curvature:

• Time-averaged intensity (or irradiance) distribution

$$W(z) = W_0 \left(1 + \left(\frac{z}{z_0}\right)^2 \right)^{1/2}$$
$$W_0 = \left(\frac{\lambda z_0}{z}\right)^{1/2}$$
$$R(z) = z \left[1 + \left(\frac{z_0}{z}\right)^2 \right]$$
$$\zeta(z) = \tan^{-1} \left(\frac{z}{z_0}\right)$$

$$I(r) = I_0 \left[\frac{W_0}{W(z)}\right]^2 \exp\left(-\frac{2\rho^2}{W^2(z)}\right)$$



• The Gaussian wave





(a) z = 0; (b) $z = z_0$; (c) $z = 2z_0$.



• The Gaussian wave

• Beam radius of a Gaussian beam

$$W(z) = W_0 [1 + (\frac{z}{z_0})^2]^{1/2},$$

W₀ is the waist radius. 2W₀ is the spot size

when
$$z \gg z_0$$
, $W(z) \approx W_0(\frac{z}{z_0}) = \theta_0 z$.



86% energy is confined in the cone.



- The Gaussian wave
 - depth of focus

$$W(z) = W_0 [1 + (\frac{z}{z_0})^2]^{1/2},$$

 W_0 is the waist radius. $2W_0$ is the spot size.



The gaussian has minimum width at z = 0.

The axial distance for the beam width $\sqrt{2}W_0$ is called depth of focus.



• The Gaussian wave

Power of a Gaussian beam

Total Power:
$$P = \int_0^\infty I(\rho, z) 2\pi \rho d\rho = \frac{1}{2} I_0 [\pi W_0]^2$$
,

The ratio of the power carried within a circle of radius ρ_0 :

$$\frac{1}{P}\int_{0}^{\rho_{0}}I(\rho,z)2\pi\rho d\rho = 1 - \exp(-\frac{2\rho_{0}^{2}}{W^{2}(z)}),$$

The power contained within a circle of radius $\rho_0 = W(z)$ \rightarrow 86% of the total power.

99% of the total power is contained within a circle of radius 1.5W(z).



Principle of superposition

 $\psi = \psi_1 + \psi_2$

• Example

Interference

 $\psi_1 = 1.0 \cos(kx), \ \psi_2 = 0.95 \cos(kx - \delta)$



Interference



Fermat's Principle and Snell's Law

- Fermat's Principle:
- The path the light takes is that for which the total travel time is minimal
- Snell's Law

 $n_1 \sin \theta_1 = n_2 \sin \theta_2$

- Total internal reflection
 - n₁>n₂

$$\theta_c = \theta_i = \arcsin\left(\frac{n_2}{n_1}\right),$$

• Evanescent waves (?)









• Fresnel's equations:



$$r_{\perp} = \frac{E_{r0,\perp}}{E_{i0,\perp}} = \frac{\cos\theta_{i} - [(\frac{n_{2}}{n_{1}})^{2} - \sin^{2}\theta_{i}]^{1/2}}{\cos\theta_{i} + [(\frac{n_{2}}{n_{1}})^{2} - \sin^{2}\theta_{i}]^{1/2}}, \quad t_{\perp} = \frac{E_{r0,\perp}}{E_{i0,\perp}} = \frac{2\cos\theta_{i}}{\cos\theta_{i} + [(\frac{n_{2}}{n_{1}})^{2} - \sin^{2}\theta_{i}]^{1/2}},$$
$$r_{\prime\prime} = \frac{E_{r0,\prime\prime}}{E_{i0,\prime\prime}} = \frac{[(\frac{n_{2}}{n_{1}})^{2} - \sin^{2}\theta_{i}]^{1/2} - (\frac{n_{2}}{n_{1}})^{2}\cos\theta_{i}}{(\frac{n_{2}}{n_{1}})^{2} - \sin^{2}\theta_{i}]^{1/2} - (\frac{n_{2}}{n_{1}})^{2}\cos\theta_{i}}, \quad t_{\prime\prime} = \frac{E_{r0,\prime\prime}}{E_{i0,\prime\prime}} = \frac{2(\frac{n_{2}}{n_{1}})\cos\theta_{i}}{(\frac{n_{2}}{n_{1}})^{2}\cos\theta_{i} + [(\frac{n_{2}}{n_{1}})^{2} - \sin^{2}\theta_{i}]^{1/2}},$$



• Fresnel's equations:



- Fresnel's equations are derived from the boundary conditions.
- phase change:

• when
$$\theta_i = 0$$
: $r_{//} = r_{\perp} = \frac{n_1 - n_2}{n_1 + n_2}$,

• At normal incidence, no phase shift if $n_1 > n_2$, 180° phase shift if $n_2 > n_1$.



• Fresnel's equations:



- Polarization angle or Brewster's angle:
 - At θ_{B} , $\theta_{B} = \arctan\left(\frac{n_{2}}{n_{1}}\right)$. $\left|r_{//}\right| = 0$. The reflected wave is linear polarized.
- Critical angle:

• At
$$\theta_c$$
, $\theta_c = \theta_i = \arcsin\left(\frac{n_2}{n_1}\right)$, $|r_{//}| = |r_{//}| = 1$. Total Internal reflection.

• The phase of the reflected wave changes with the incident angle.



• Fresnel's equations:



- Total internal reflection and evanescent wave
 - Evanescence wave: $E_{\perp} = e^{-\alpha y} \exp(j\omega t k_z z),$ $\alpha = [k_{2,z}^2 - k_2^2]^{1/2} = \frac{2\pi}{\lambda} [\sin^2 \theta_i - (\frac{n_2}{n_1})^2]^{1/2},$
 - Penetration depth:

 $\delta = 1/\alpha$,



- Fresnel's equations:
 - Reflectance and Transmittance

$$R_{\perp} = \frac{\left|E_{r0,\perp}\right|^{2}}{\left|E_{i0,\perp}\right|^{2}} = \left|r_{\perp}\right|^{2}, \qquad \qquad R_{\prime\prime} = \frac{\left|E_{r0,\prime\prime}\right|^{2}}{\left|E_{i0,\prime\prime}\right|^{2}} = \left|r_{\prime\prime}\right|^{2},$$

$$T_{\perp} = \frac{n_2}{n_1} \frac{\left| E_{t0,\perp} \right|^2}{\left| E_{i0,\perp} \right|^2} = \frac{n_2}{n_1} \left| t_{\perp} \right|^2, \qquad T_{\prime\prime} = \frac{n_2}{n_1} \frac{\left| E_{t0,\prime\prime} \right|^2}{\left| E_{i0,\prime\prime} \right|^2} = \frac{n_2}{n_1} \left| t_{\prime\prime} \right|^2,$$

• Normal Incidence

$$R_{\perp} = R_{//} = (\frac{n_1 - n_2}{n_1 + n_2})^2, \qquad T_{\perp} = T_{//} = \frac{4n_1n_2}{(n_1 + n_2)^2},$$

Polychromatic waves



Polychromatic waves

$$E(r,t) = \int_{-\infty}^{\infty} A(\omega)e^{i(k(\omega)\cdot r-\omega t)}d\omega, \qquad k(\omega) = k_{\omega_0} + \frac{dk}{d\omega}|_{\omega_0} (\omega - \omega_0) + ...,$$
$$\Rightarrow E(r,t) = \int_{-\infty}^{\infty} A(\omega)e^{i(k(\omega)\cdot r-\omega t)}d\omega = 2A(\omega_0)\Delta\omega\frac{\sin\psi}{\psi}e^{i(k_0\cdot r-\omega t)}, \qquad \psi = [\frac{d\omega}{dk}|_{\omega_0} r-t]\Delta\omega,$$

Group velocity

• Wave packet speed: $v_g = \frac{d\omega}{dk}|_{\omega_0}$,

$$\begin{split} v_{g} = & \frac{d\omega}{dk} |_{\omega_{0}} = \frac{c}{n - \lambda} \frac{dn}{d\lambda}, \\ N_{g} = & n - \lambda \frac{dn}{d\lambda} \text{ is called group index} \end{split}$$



• Exercise: the difference between phase velocity and group velocity?

Polychromatic waves



Absorption and dispersion

 $E(r,t) = Ae^{i(k \cdot r - \omega t)}, \qquad k(\omega) = k_0 n = k_0 (1+\chi)^{1/2} = k_0 (1+\chi' + i\chi'')^{1/2} = \beta - i\frac{1}{2}\alpha,$

Absorption or attenuation coefficient

α,

- Dispersive media = Velocity depends on frequency $\beta = nk_0$,
- Wave packet broadening for a length of L:

$$\Delta T = \frac{d}{d\omega} \left(\frac{L}{v_g}\right) \Delta \omega = L \frac{d^2 k}{d\omega^2} \Delta \omega,$$

• Dispersion parameter:

$$D = \frac{d}{d\lambda} \left(\frac{1}{v_g}\right) = -\frac{2\pi c}{\lambda^2} \frac{d^2 k}{d\omega^2} (ps/(km - nm)),$$



Optical Fibers

- An optical fiber is essentially a waveguide for light
- It consists of a core and cladding that surrounds the core
- The index of refraction of the cladding is less than that of the core, causing rays of light leaving the core to be refracted back into the core
- Advantages of optical fiber include:
 - Greater bandwidth than copper
 - Lower loss
 - Immunity to crosstalk
 - No electrical hazard







Optical Fibers & Communications





(a) Fiber cross section



Optical Fibers



- Optical fiber is made from thin strands of either glass or plastic
- It has little mechanical strength, so it must be enclosed in a protective jacket
- Often, two or more fibers are enclosed in the same cable for increased bandwidth and redundancy in case one of the fibers breaks
- It is also easier to build a full-duplex system using two fibers, one for transmission in each direction



Total Internal Reflection



- Optical fibers work on the principle of total internal reflection
- With light, the refractive index is listed
- The angle of refraction at the interface between two media is governed by Snell's law:

 $n_1\sin\theta_1 = n_2\sin\theta_2$





Numerical Aperture



- The numerical aperture of the fiber is closely related to the critical angle and is often used in the specification for optical fiber and the components that work with it
- The numerical aperture is given by the formula:

$$N.A. = \sqrt{n_1^2 - n_2^2}$$

• The angle of acceptance is twice that given by the numerical aperture



Modes and Materials



Optical fiber = waveguide

- \rightarrow light can propagate in a number of modes
- Large diameter
 - Light entering at different angles → excites different modes
- Small diameter
 - May only excite one mode
- Multimode propagation
 - Will cause dispersion \rightarrow Spreading of • pulses \rightarrow limits the usable bandwidth

Single-mode fiber

- Much less dispersion, but ...
- More expensive to produce
- More difficult to couple to light sources
 - Small size
 - Smaller numerical aperture







| = 2, m = 2



| = 1, m = 3



V-Number and Fiber Modes





Figure 3.10. Normalized propagation constant, b, for designated LP modes as functions of V.

$$V = 2\pi a (NA) / \lambda$$

a: radius of core
 λ : wavelength of ligth

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Cut-off Wavelength

Definition:

- The wavelength below which multiple modes of light can be propagated along a particular fiber, i.e.,
 - $\lambda \ge \lambda_c$, single mode
 - $\lambda < \lambda_c$, multi-mode

 $\lambda_c = \frac{2\pi a}{2.405} \times NA$



Multi-Mode vs. Single-mode





Types of Fibers – Step Index



Radial index distribution $n(r) = \begin{cases} n_1 & r < a \\ \\ n_2 & r \ge a \end{cases}$

Typical values $\Delta = 1\% - 1\%$

n =1.44 - 1.46



Types of Fibers – Step Index



Radial index distribution

$$n(r) = \begin{cases} n_1 \left[1 - \Delta \left(\frac{r}{a}\right)^x \right] & r < a \\ n_2 & r \ge a \end{cases}$$

Graded-index fiber has less dispersion than a multimode step-index fiber



Dispersion



- Dispersion in fiber optics results from the fact that in multimode propagation, the signal travels faster in some modes than it would in others
- Single-mode fibers are relatively free from dispersion except for *intramodal dispersion*
- Graded-index fibers reduce dispersion by taking advantage of higher-order modes
- One form of intramodal dispersion is called *material dispersion* because it depends upon the material of the core
- Another form of dispersion is called *waveguide dispersion*
- Dispersion increases with the bandwidth of the light source



Losses



- Losses in optical fiber result from attenuation in the material itself and from scattering, which causes some light to strike the cladding at less than the critical angle
- Bending the optical fiber too sharply can also cause losses by causing some of the light to meet the cladding at less than the critical angle
- Losses vary greatly depending upon the type of fiber
 - Plastic fiber may have losses of several hundred dB per kilometer
 - Graded-index multimode glass fiber has a loss of about 2–4 dB per kilometer
 - Single-mode fiber has a loss of 0.4 dB/km or less



Splices and Connectors



- In fiber-optic systems, the losses from splices and connections can be more than in the cable itself
- Losses result from:
 - Axial or angular misalignment
 - Air gaps between the fibers
 - Rough surfaces at the ends of the fibers



Fiber-Optic Connectors



- Coupling the fiber to sources and detectors creates losses as well, especially when it involves mismatches in numerical aperture or in the size of optical fibers
- Good connections are more critical with single-mode fiber, due to its smaller diameter and numerical aperture
- A *splice* is a permanent connection and a *connector* is removable



Optical Couplers and Switches



- As with coaxial cable and microwave waveguides, it is possible to build power splitters and directional couplers for fiber-optic systems
- It is more complex and expensive to do this with fiber than with copper wire
- Optical couplers are categorized as either star couples with multiple inputs and outputs or as tees, which have one input and two outputs
- Optical couplers can be made in many different ways:
 - A number of fibers can be fused together to make a transmissive coupler
 - A reflective coupler allows a signal entering on any fiber to exit on all other fibers, so the coupler is bidirectional





Optical Switches and Relays

- Occasionally, it is necessary to switch optical signals from one fiber to another
- The simplest type of optical switch moves fibers so that an input fiber can be positioned next to the appropriate output fiber
- Another approach is direct the incoming light into a prism, which reflects it into the outgoing fiber. By moving the prism, the light can be switched between different output fibers
- Lenses are necessary with this approach to avoid excessive loss of light







Ports 2 and 3 connected together





Other material



- Lenses will be covered in the imaging lecture
- Coherence will be covered in the OCT lecture.

