ECE 370
Introduction to Biomedical Engineering

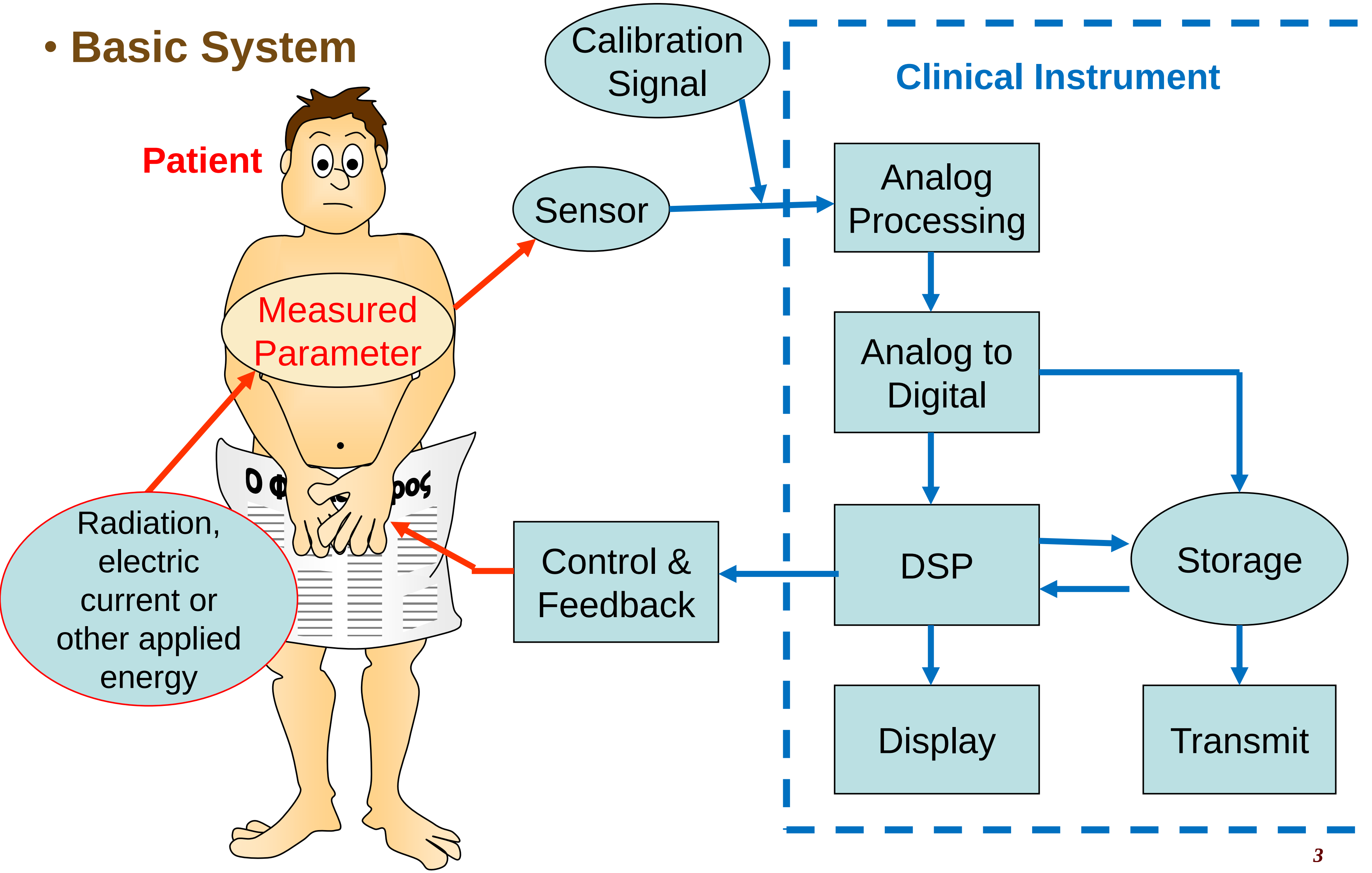
Bioinstrumentation - Bioelectronics

Biomedical Instrumentation

- **Diagnosis and therapy depend heavily on the use of medical instrumentation.**
- **Medical procedures:**
  - Medicine can be defined as a multistep procedure on an individual by a physician or group of physicians repeated until the symptoms disappear
- **Medical Procedure**
  1. Collection of data - qualitative and/or quantitative
  2. Analysis of data
  3. Decision making
  4. Treatment planning based on the decision
- **Medical devices play a critical role in all of the above steps**
  - Diagnostic, Therapeutic, Monitoring devices

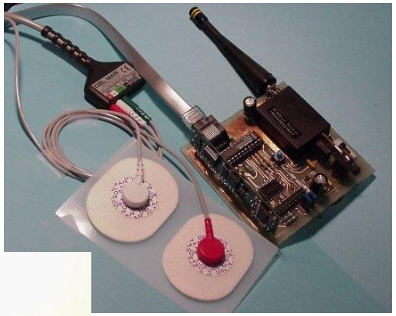
Biomedical Instrumentation

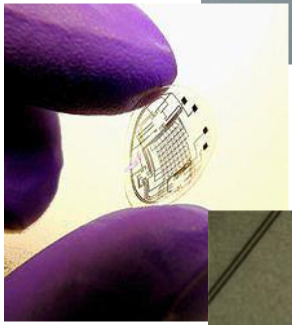
- **Basic System**

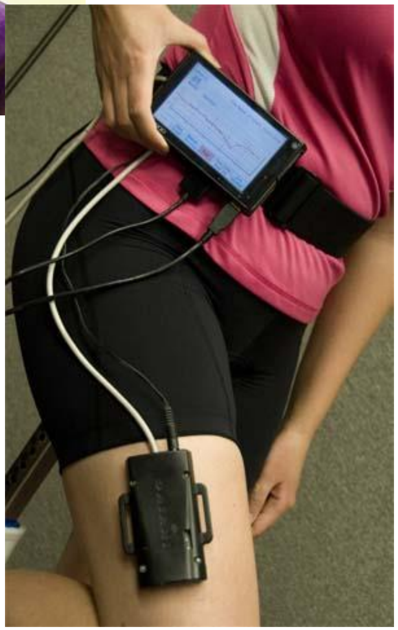


Biomedical Instrumentation

- **Components of Biomedical Instrumentation System…**
- **A sensor**
  - Detects biochemical, bioelectrical, or biophysical parameters
  - Provides a safe interface with biological materials
  - Converts a physical parameter to an electrical output
    - Should respond only to the form of energy present in the parameter
    - Should be minimally invasive (ideally noninvasive)
- **Electronics**
  - Signal conditioning
    - Amplify, filter, match the impedance of the sensor
  - Further analog processing
  - Convert analog signal to digital
  - The electronics interface must
    - Match the electrical characteristics of the sensor/actuator with computation unit
    - Preserve the signal to noise ratio of sensor
    - Preserve the bandwidth (i.e., time response) of sensor/actuator
    - Provides a safe interface with the sensor/actuator
    - Provides a safe interface with the computation unit
- **An actuator**
  - Delivers external agents via direct or indirect contact
  - Controls biochemical, bioelectrical, or biophysical parameters
  - Provides a safe interface with biologic materials







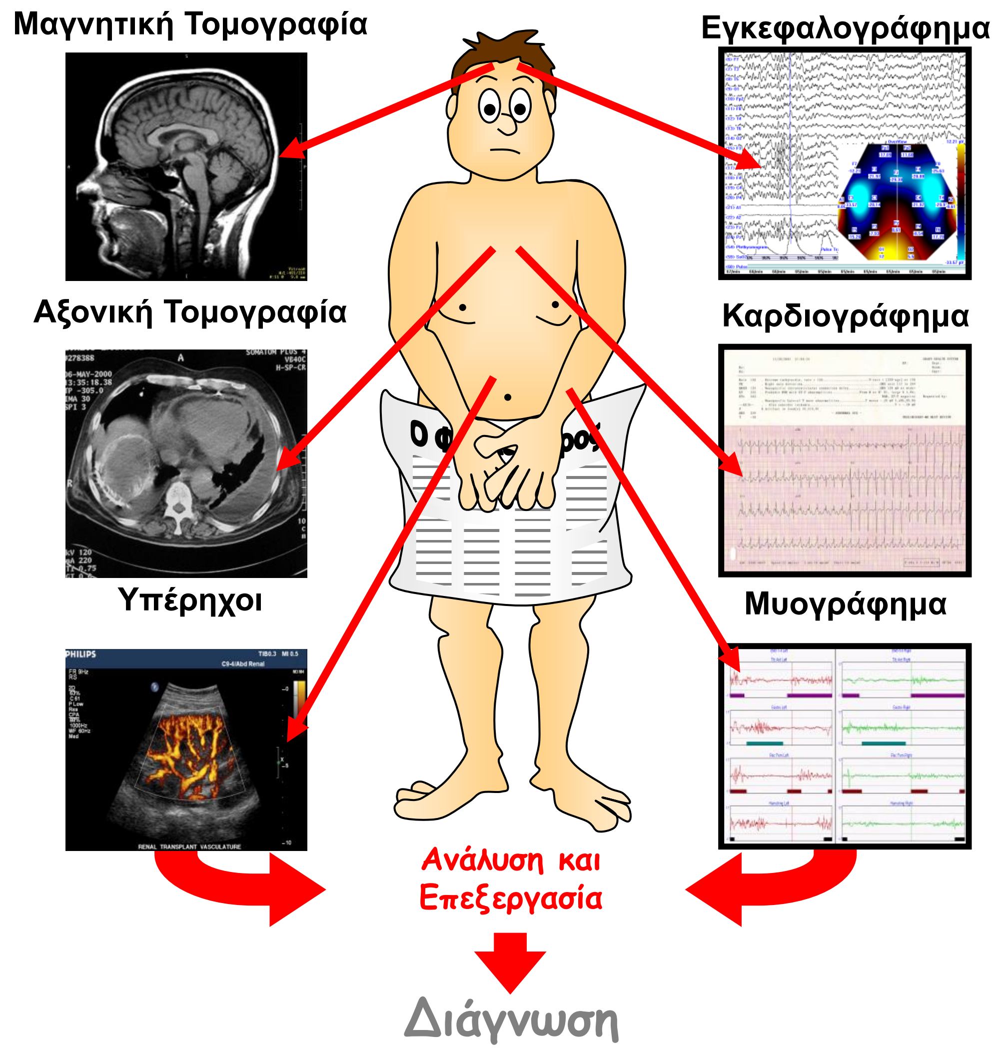
Biomedical Instrumentation

Components of Biomedical Instrumentation System…

- **The computation unit**
  - provides primary user interface
  - provides primary control for the overall system
  - provides data storage for the system
  - provides primary signal processing functions for the system
  - maintains safe operation of the overall system
- **Output (display or printer)**
  - Results must be displayed in a form that the human operator can perceive
    - Numerical, Graphical, Discrete or continuous, Permanent or temporary, Visual or acoustical
- **Auxiliary elements**
  - Data storage
  - Data transmission
  - Control and feedback
  - Calibration signal

Biomedical Instrumentation

- **Measured Parameters**
  - Physical quantity, property, or condition that the system measures
    - Biopotential
    - Pressure
    - Flow
    - Dimension (imaging)
    - Displacement (velocity, acceleration, and force)
    - Impedance
    - Temperature
    - Chemical concentrations



Biomedical Instrumentation

- **Measured Parameters**

Measurement	**Range**	**Frequency, Hz**	**Method**
Blood flow	1 to 300 mL/s	0 to 20	Electromagnetic or ultrasonic
Blood pressure	0 to 400 mmHg	0 to 50	Cuff or strain gage
Cardiac output	4 to 25 L/min	0 to 20	Fick, dye dilution
Electrocardiography	0.5 to 4 mV	0.05 to 150	Skin electrodes
Electroencephalography	5 to 300 μ V	0.5 to 150	Scalp electrodes
Electromyography	0.1 to 5 mV	0 to 10000	Needle electrodes
Electroretinography	0 to 900 μ V	0 to 50	Contact lens electrodes
pH	3 to 13 pH units	0 to 1	pH electrode
pCO_2	40 to 100 mmHg	0 to 2	pCO_2 electrode
pO_2	30 to 100 mmHg	0 to 2	pO_2 electrode
Pneumotachography	0 to 600 L/min	0 to 40	Pneumotachometer
Respiratory rate	2 to 50 breaths/min	0.1 to 10	Impedance
Temperature	32 to 40 °C	0 to 0.1	Thermistor

Biomedical Instrumentation

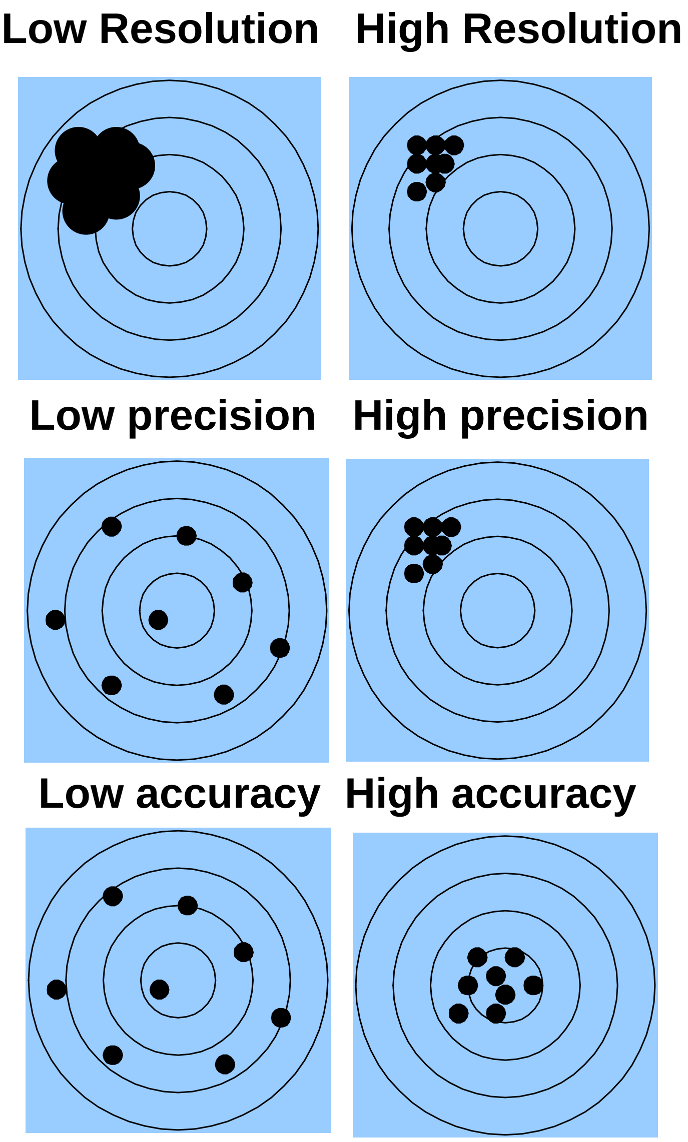
- **Bioinstrumentation should be designed with a specific signal in mind.**
  - The values of the specifications
    - Have been agreed upon by committees
    - Are drawn from research, hospitals, industry, and government.

Specifications for an electrocardiograph

Specification	**Value**
Input signal dynamic range	±5 mV
Dc offset voltage	±300 mV
Slew rate	320 mV/s
Frequency response	0.05 to 150 Hz
Input impedance at 10 Hz	2.5 MΩ
Dc lead current	0.1 μA
Return time after lead switch	1 s
Overload voltage without damage	5000 V
Risk current at 120 V	10 μA

Biomedical Instrumentation

- **Biomedical measurement systems should demonstrate high …**
- **Resolution**
  - The smallest incremental quantity that can be reliably measured.
  - However, high resolution does not imply high accuracy.
- **Precision**
  - Obtaining the same value from repeated measurements from the same input under the same conditions.
  - However, high precision does not imply high accuracy
- **Accuracy**
  - Obtaining the true value from repeated measurements from the same input under the same conditions
- **Repeatability**
  - The quality of obtaining the same output from repeated measurements from the same input over a period of time.



Biomedical Instrumentation

Biomedical measurement systems should also demonstrate high …

- **Sensitivity**
  - Ability to distinguish between “normal” and “abnormal” results, “safe” and “unsafe” levels
- **Non-Intrusivity**
  - Does not interfere or alter the function being performed and measured
  - Does not interfere with the normal function of the body (ideally non-invasive) except if used for therapy!
- **Safety**
  - Should not pose danger to subject or researcher
- **Practicality**
  - Simple, easy, and inexpensive to use
  - Minimize possibility of errors

Biomedical Instrumentation

- **Problems encountered in measuring a living system**
  - Many crucial variables in living systems are inaccessible and can not be directly measured
  - Variables measured are seldom deterministic
    - Intra-patient and Inter-patient variability
  - Some biomedical measurements depend on the energy of an external stimulus
  - Operation of instruments in the medical environment imposes important additional constraints

Biomedical Instrumentation

- **Errors in measurements**
  - When we measure a variable, we seek to determine the true value
  - This true value may be corrupted by a variety of errors.
  - For example
    - Electric and magnetic fields from the power lines may couple into the wires and cause an undesired added voltage called interference
    - Movement of electrodes on the skin may cause an undesired added voltage called an artifact.
  - Electronic noise
    - Thermal voltages in the amplifier semiconductor junctions may cause undesired added random voltage changes
  - Drift
    - Temperature changes in the amplifier electronic components may cause undesired slow changes in voltage
  - Digitization errors
    - Measurements become discrete (steps) in both time and amplitude
- **We must evaluate each of these error sources to determine their size and what we can do to minimize them**
  - e.g. frequency filters can be used to reduce noise and interference.

Biomedical Instrumentation

- **Calibration**
  - The instruments should be calibrated against a standard
    - An accuracy 3 to 10 times better than the desired calibration accuracy
  - The accuracy of the standard should be traceable
    - National Institute of Standards and Technology, TSI, etc.)
- **Certification**
  - The instruments should be certified for safety
    - E.g. CE
- **Regulatory Approval**
  - The instruments must be tested in clinical trials
  - Their clinical use must be approved by the appropriate bodies
    - E.g. FDA

Amplification Basics

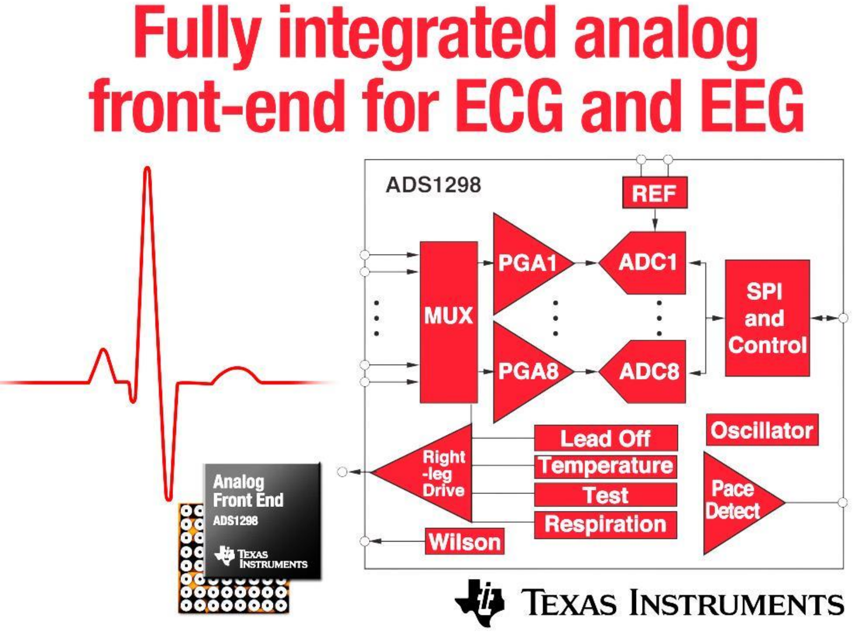
- **The term "amplify" basically means to make stronger.**
- **Types of amplification**
  - There are three kinds of amplifications: Two major types, and the third type is derived from the another two :
    - Voltage Amplifier - an amp that boosts the voltage of an input signal
    - Current Amplifier - an amp that boosts the current of a signal
    - Power Amplifier - the combination of the above two amplifiers

Bioelectric Amplifier

- **Bioelectric Amplifier**
  - Is the amplifier that used to process bio-potentials
  - The gain may be
    - Low gain amp: gain factor bw X1 and X10 (EX: action potential)
    - Medium gain amp: gain factor bw X10 and X1000 (EX: ECG, Muscles potentials, …)
    - High gain or low-level signal amp: gain factor over X10000 to as high as X1000000 (EX: EEG)
  - It is usually ac coupled.
  - DC-coupling is required where the input signal are clearly dc or change very slowly (0.05 Hz)
  - Exceptional for EX.: ECG signal should be AC coupled despite of the component as low as 0.05 Hz to overcome electrode offset potential from electrode-skin connection
  - The high-frequency response is the frequency at which the gain drops 3dB below its midfrequency value (for ECG form 0.05 to 100 Hz)

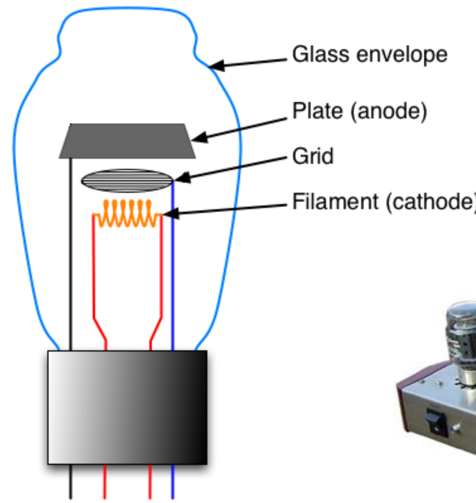
Bioelectric Amplifier

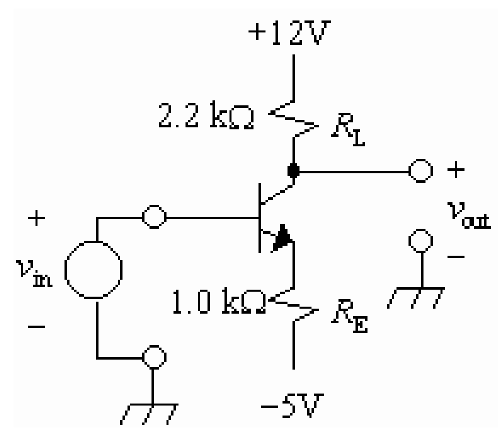
- **Important parameters in a bioelectric amp:**
  - **Noise**: normally is the thermal noise generated in resistances and semiconductors devices.
  - **Drift**: change in output signal voltage caused by change in operating temperature.
  - **High input impedance**: 10^7 to 10^{12} Ω and it should be at least an order of magnitude high than the source impedance.
  - **Integrated circuit** (IC) operational amplifier is well suited as bioelectric amp because of these properties.



Amplification Basics

- **Types of Amplifiers**
  - Vacuum Valve
  - Transistor
  - Operational amplifier
- **Vacuum Tube**
  - In electronics, a vacuum tube or (outside North America) thermionic valve or just valve, is a device generally used to amplify, switch or otherwise modify, a signal by controlling the movement of electrons in an evacuated space.
- **Transistor**
  - Bipolar junction transistor (BJT) are two diodes joined with a very thin common region
  - A small electrical input can be amplified by transistor

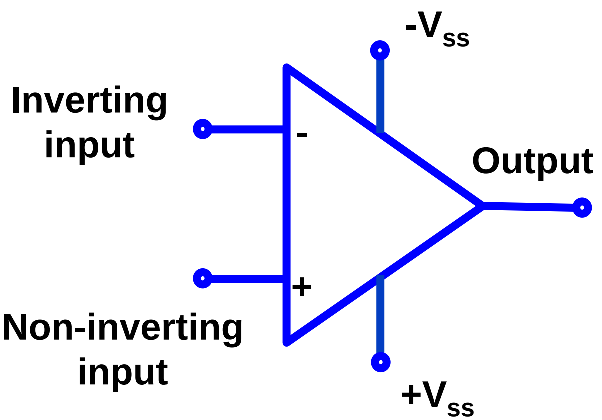




A simple one-transistor amplifier with positive and negative supplies

Operational Amplifier (Op-amp)

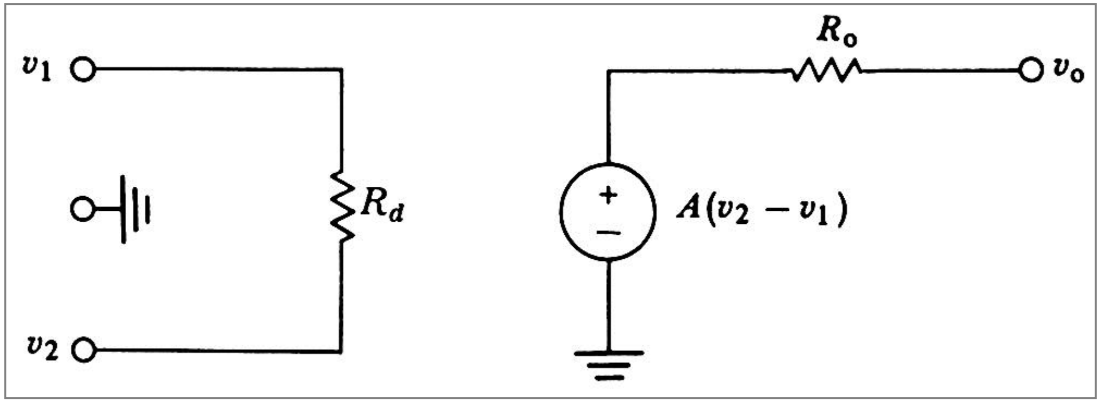
- **The op-amp is a device for increasing the power of a signal.**
  - It does this by taking power from a power supply and controlling the output to match the input signal shape but with a larger amplitude (Amplification).
- **The op-amp is used also to perform arithmetic operations (addition, subtraction, multiplication) with signals.**
- **The properties of the negative feedback loop determine the properties of the circuit containing an op-amp.**
- **It has two inputs: the inverting input (-) and the non-inverting input (+), and one output.**
- **It has usually two supplies (±Vss) but it can work with one.**



Symbol of op-amplifier

Operational Amplifier (Op-amp)

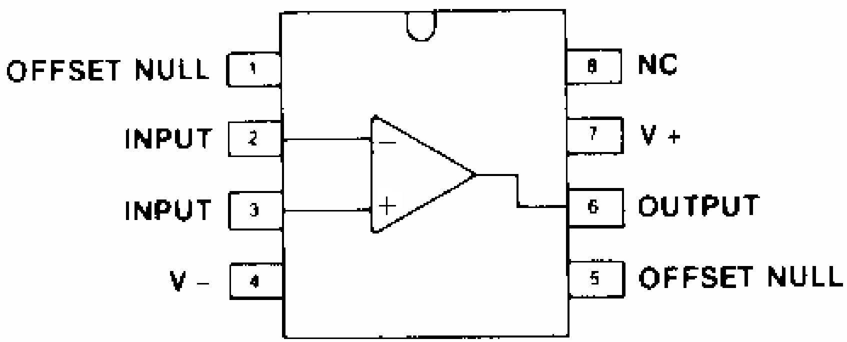
- **Ideal Op-Amp**
  - Op-amp equivalent circuit:



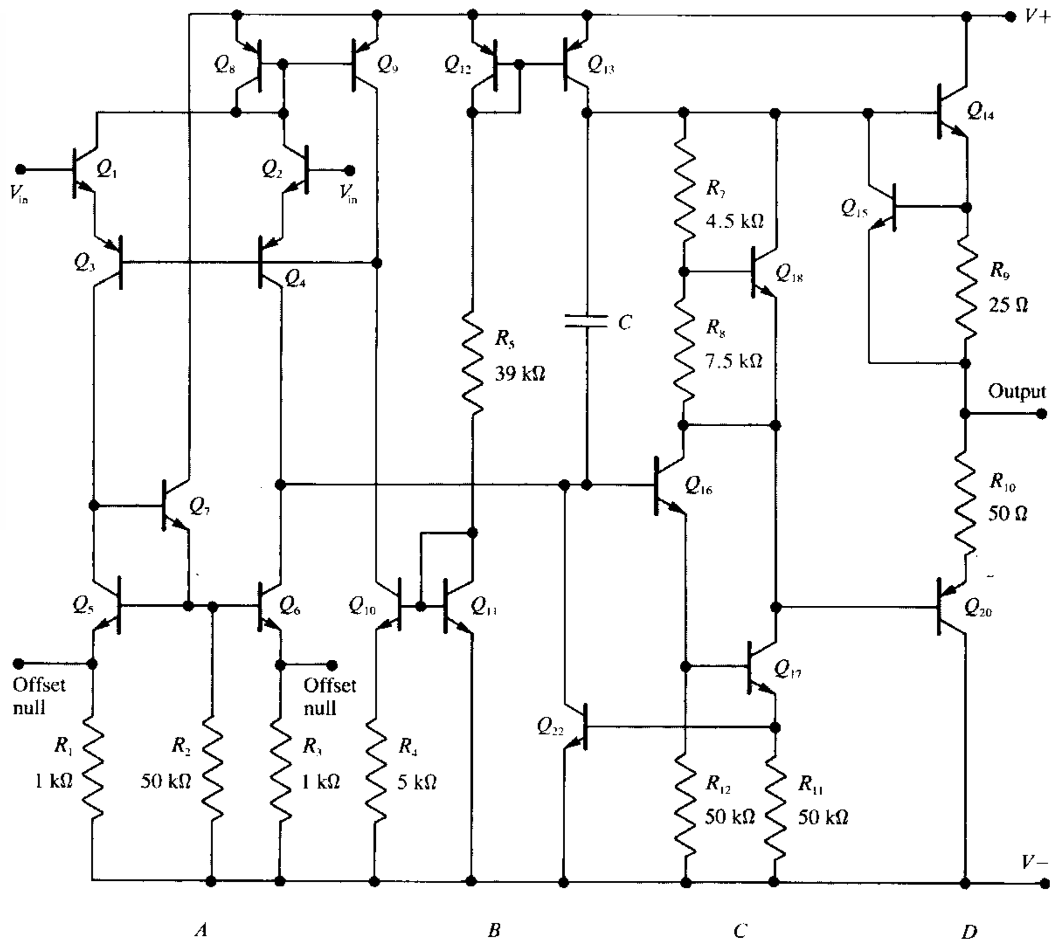
  - The two inputs are υ1 and υ 2.
  - A differential voltage between them causes current flow through the differential resistance Rd.
  - The differential voltage is multiplied by A (the open-loop gain of the op amp) to generate the output-voltage source
  - Any current flowing to the output terminal vo must pass through the output resistance Ro.

Operational Amplifier (Op-amp)

- **Inside the Op-Amp (IC-chip)**



741 op amp



20 transistors
11 resistors
1 capacitor

Operational Amplifier (Op-amp)

- **Real vs. Ideal Op-amp**

Parameter	Ideal Op Amp	Typical Op Amp
Open-loop voltage gain **A**	∞	$10^5 – 10^9$
Common mode voltage gain	0	10^{-5}
Frequency response **f**	∞	1- 20 MHz
Input impedance $\mathbf{Z_{in}}$	∞	10^6 Ω (bipolar) $10^9–10^{12}$ Ω (FET)
Output impedance $\mathbf{Z_{out}}$	0	100 – 1000 Ω

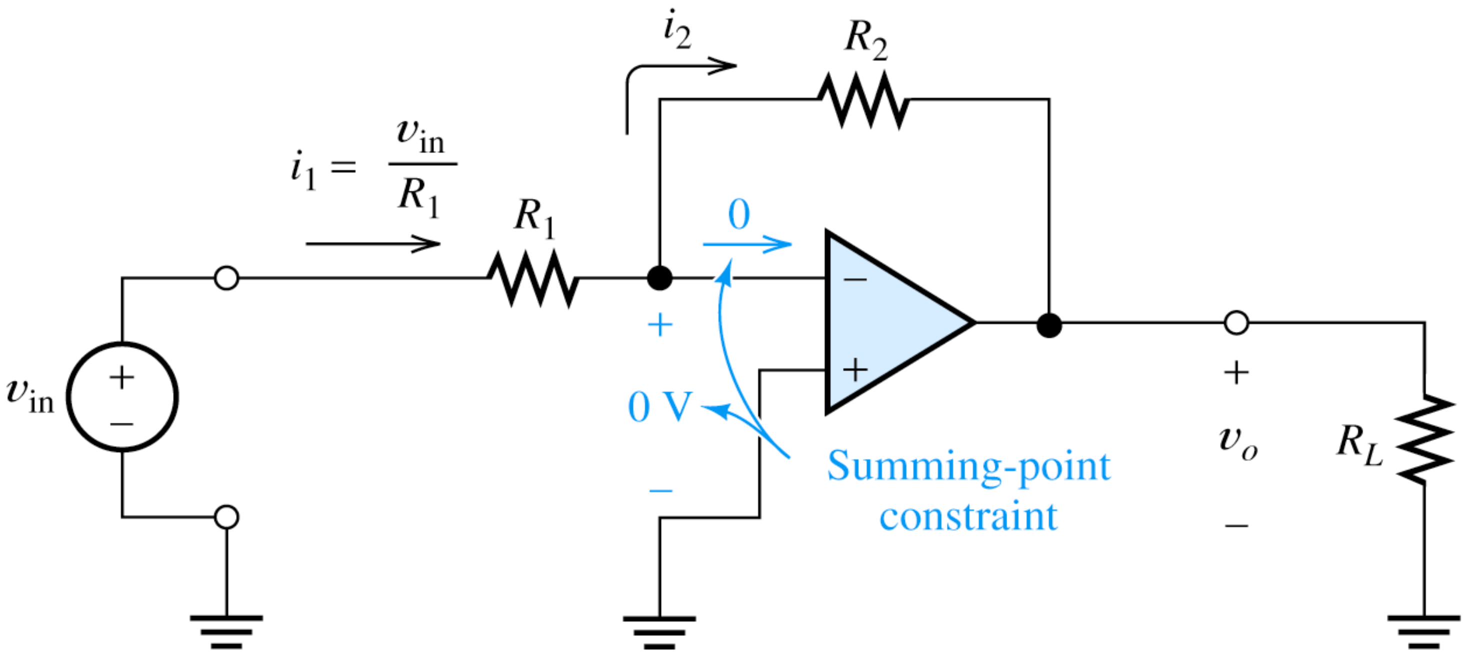
Operational Amplifier (Op-amp)

- **Summing Point Constraint**
  - In a negative feedback system, the ideal op-amp output voltage attains the value needed to force the differential input voltage and input current to zero.

- **Circuit solution**
  1. Verify that negative feedback is present.
  2. Assume that the differential input voltage and the input current of the op amp are forced to zero. (This is the summing-point constraint.)
  3. Apply standard circuit-analysis principles, such as Kirchhoff's laws and Ohm's law, to solve for the quantities of interest.

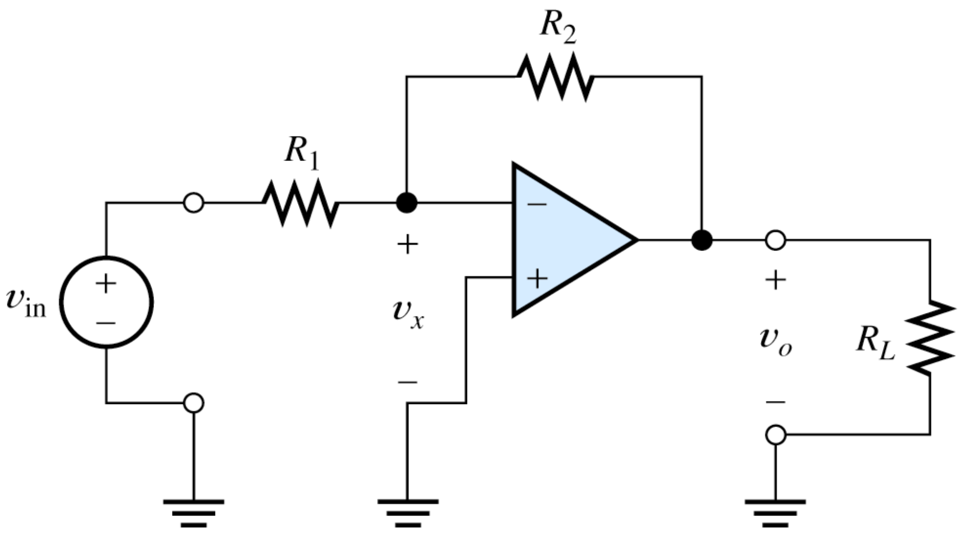
Operational Amplifier (Op-amp)

- **Applying the Summing Point Constraint**



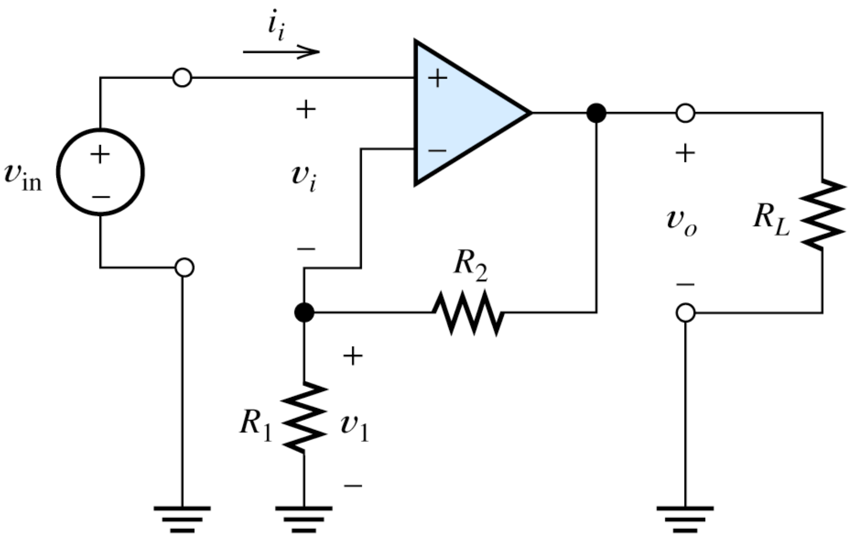
Operational Amplifier (Op-amp)

- **Inverting Amplifier**



$$A_v = \frac{v_o}{v_{\text{in}}} = -\frac{R_2}{R_1}$$

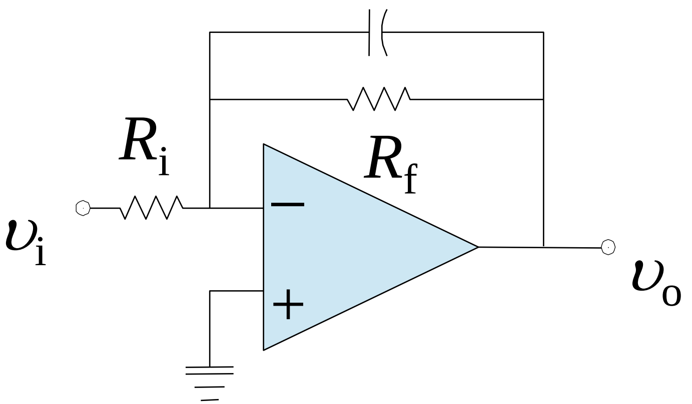
- **Non-inverting Amplifiers**

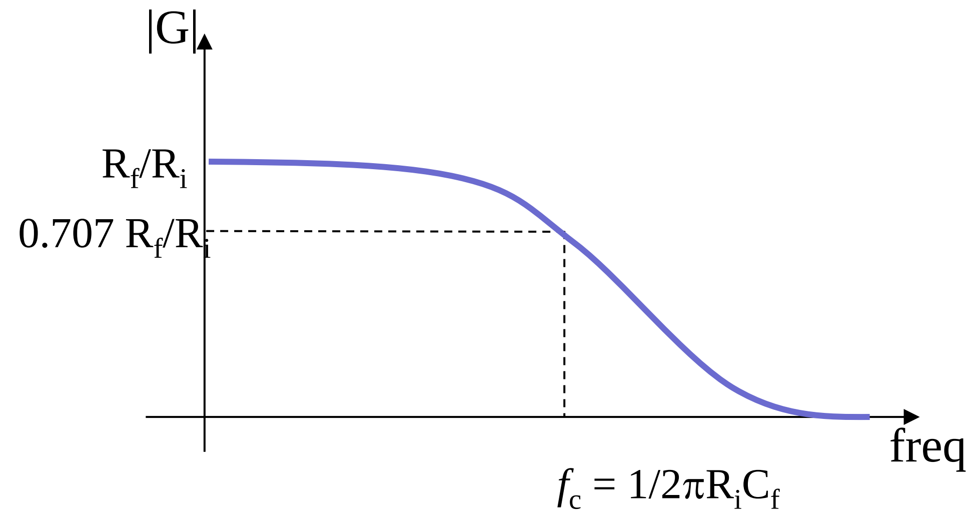


$$A_v = \frac{v_o}{v_{\text{in}}} = 1 + \frac{R_2}{R_1}$$

Operational Amplifier (Op-amp)

- **Active Filters- Low-Pass Filter**
  - A low-pass filter attenuates high frequencies

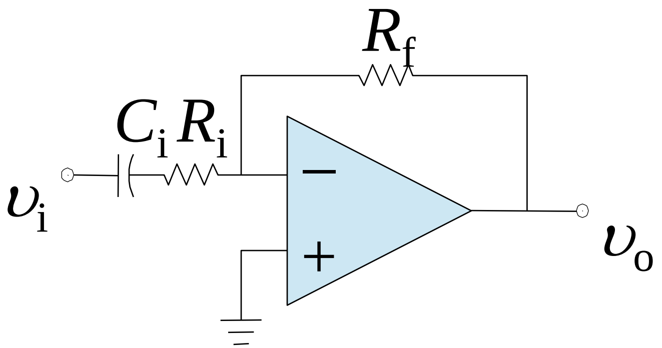


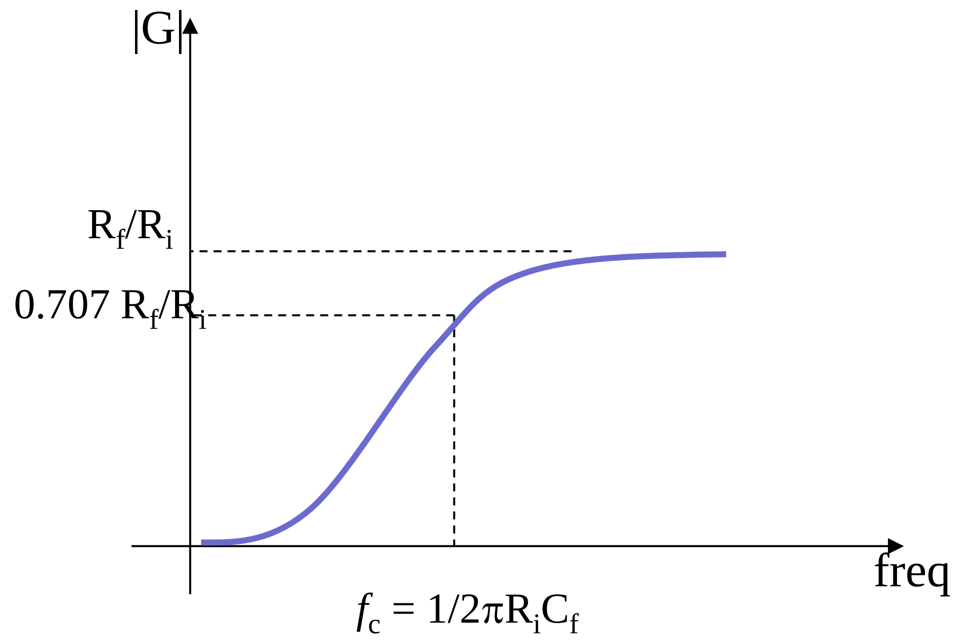


$$\text{Gain} = G = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{-R_f}{R_i}\frac{1}{1 + j\omega R_f C_f}$$

Operational Amplifier (Op-amp)

- **Active Filters (High-Pass Filter)**
  - A high-pass filter attenuates low frequencies and blocks dc.

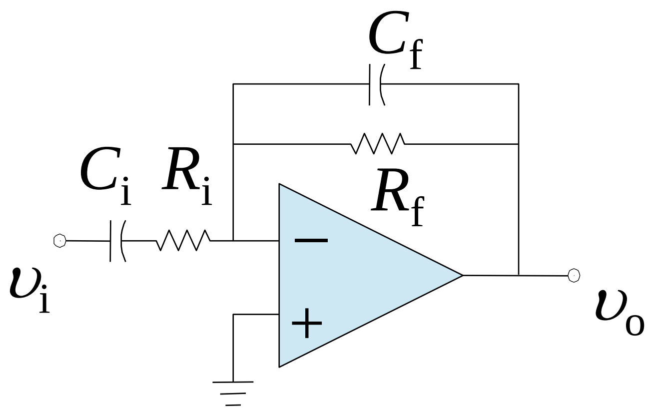


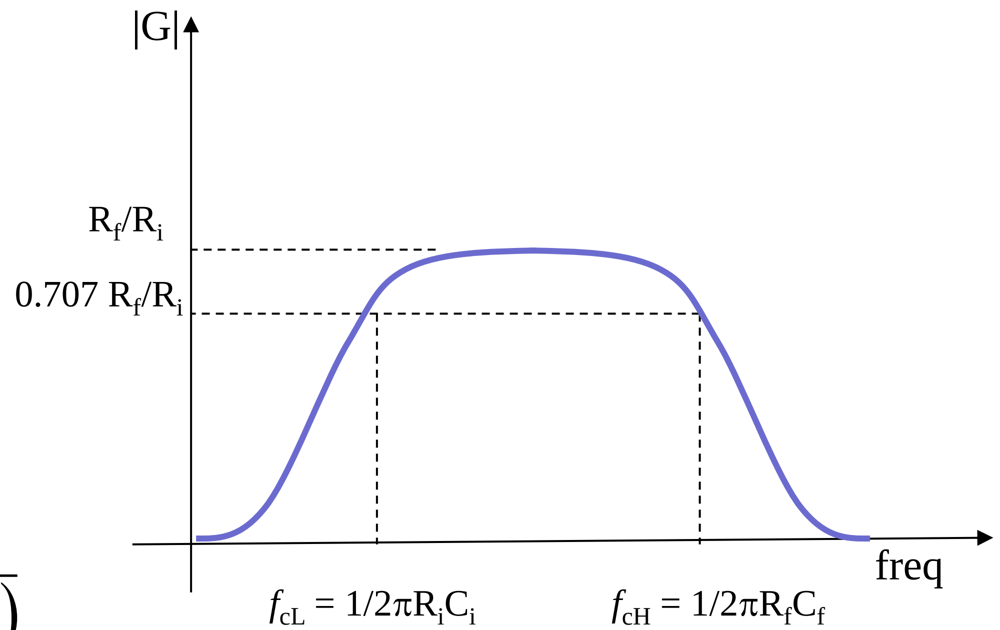


$$\text{Gain} = \text{G} = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{-R_f}{R_i}\frac{j\omega R_i C_i}{1 + j\omega R_i C_i}$$

Operational Amplifier (Op-amp)

- **Active Filters (Band-Pass Filter)**
  - A bandpass filter attenuates both low and high frequencies.

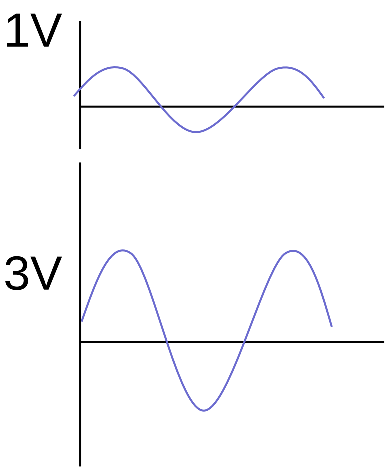


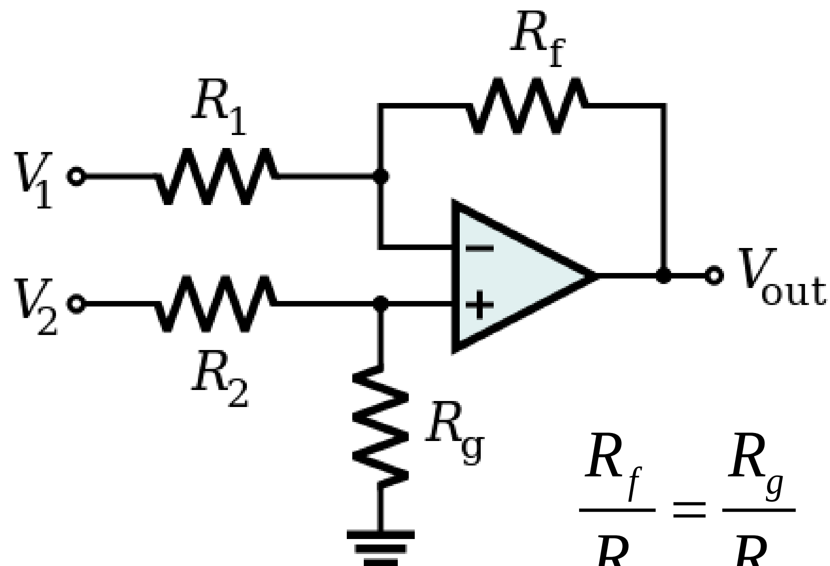


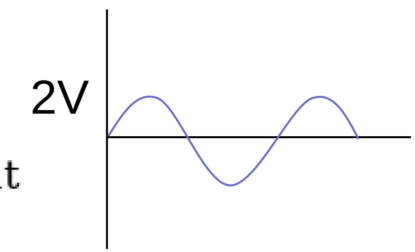
$$\frac{V_o(j\omega)}{V_i(j\omega)} = \frac{-j\omega R_f C_i}{(1 + j\omega R_f C_f)(1 + j\omega R_i C_i)}$$

Operational Amplifier (Op-amp)

- **Differential Amplifier**
  - In differential mode you can signals common to both input signals





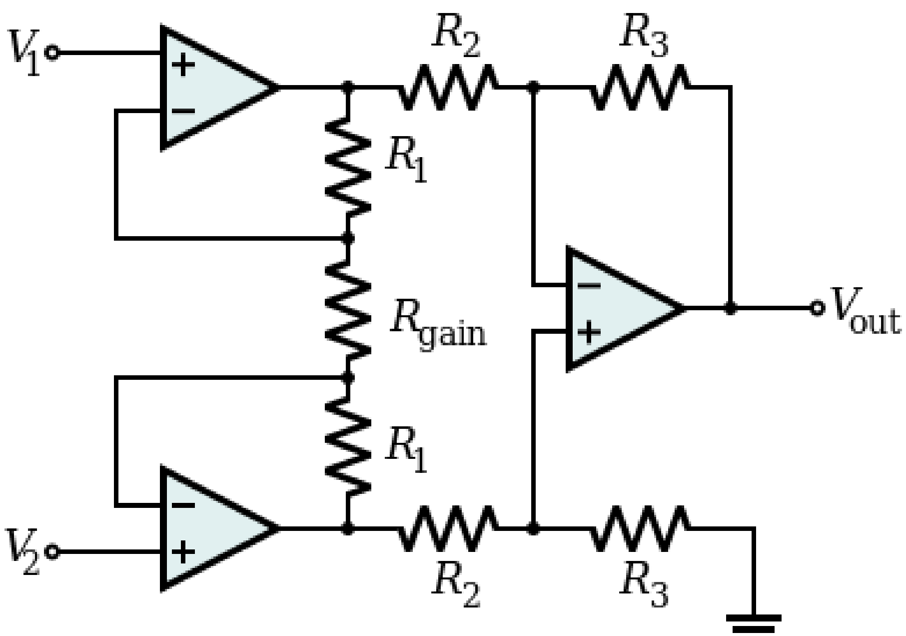


$$\frac{R_f}{R_1} = \frac{R_g}{R_2}$$

$$V_{out} = \frac{R_f}{R_1}\left(V_2 - V_1\right)$$

Operational Amplifier (Op-amp)

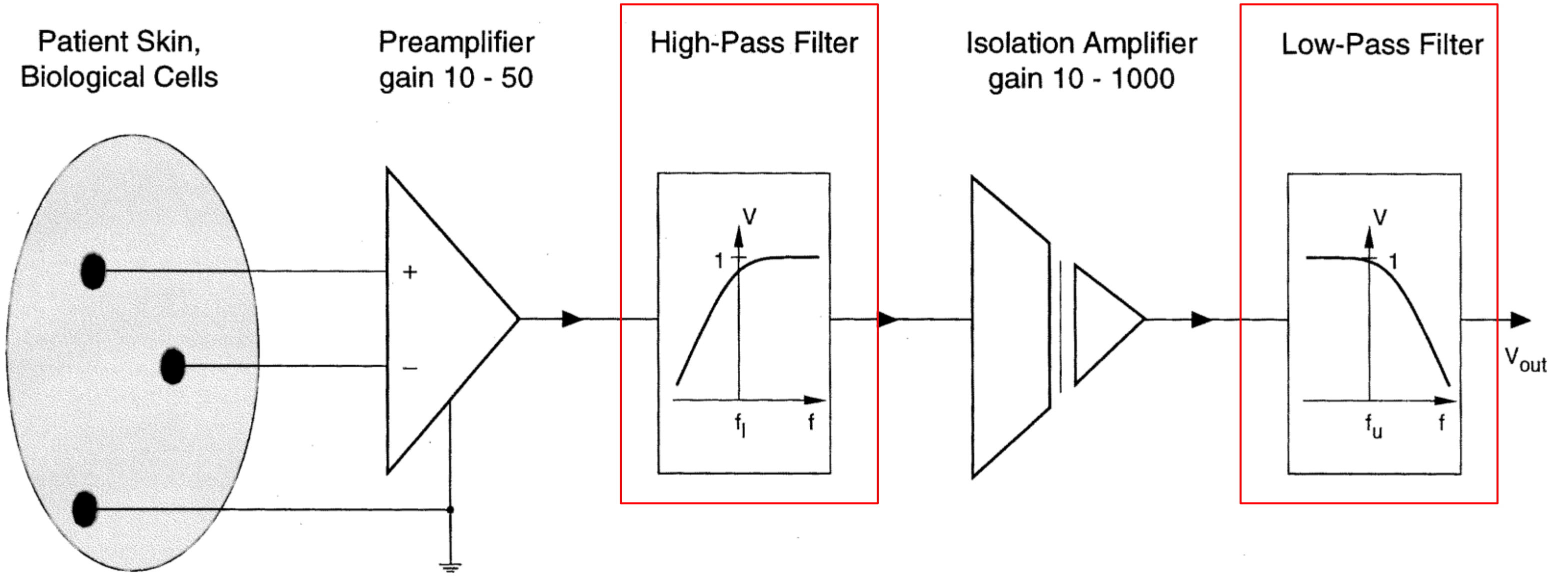
- **Instrumentation Amplifier**
  - High gain and high-input impedance.
  - Composed of 2 amplifiers in noninverting format and a 3rd amplifier as a differential amplifier



$$\frac{V_{out}}{V_2 - V_1} = \left(1 + \frac{2R_1}{R_{gain}}\right)\left(\frac{R_3}{R_2}\right)$$

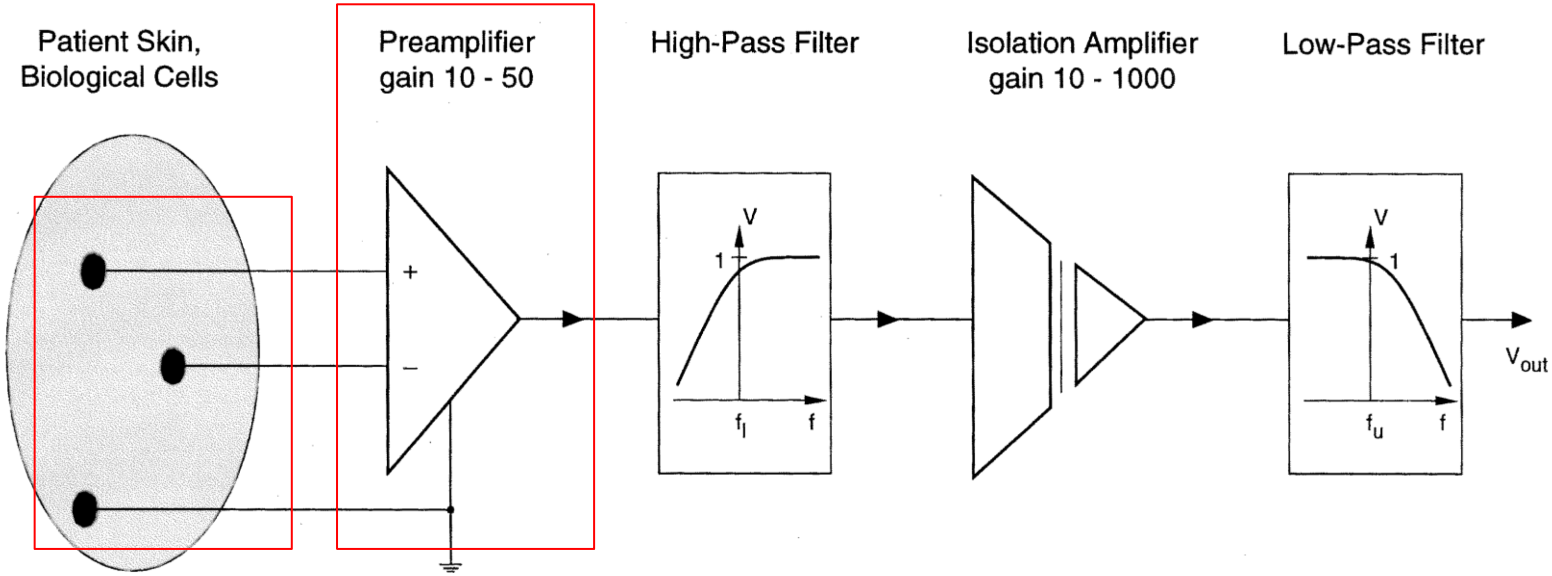
Operational Amplifier (Op-amp)

- **Additional considerations in a bioamplifier**
  - Noise
    - DC Drift (low frequency)
    - Electronic noise (low and high frequency)
    - Line interference (50 or 60 Hz)
  - How to decrease noise artifacts?
    - Shielding of electrodes and equipment
    - Filtering (Low pass, high pass)



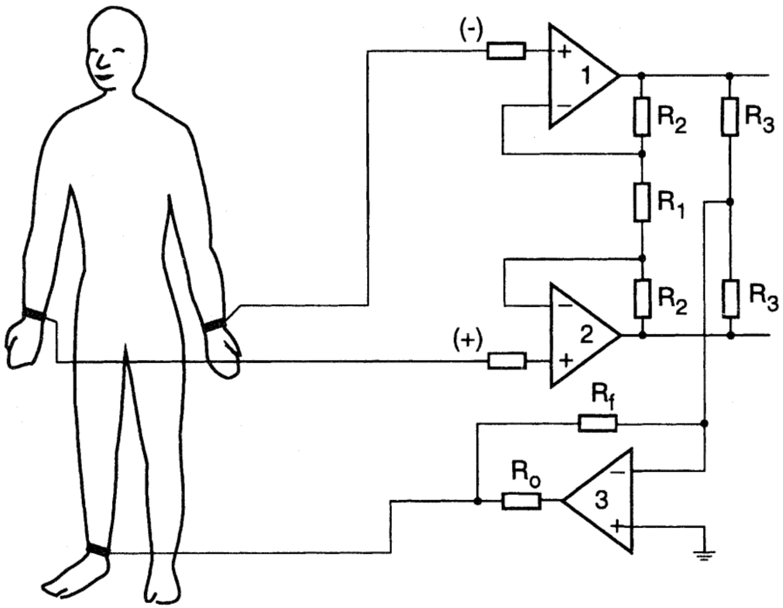
Operational Amplifier (Op-amp)

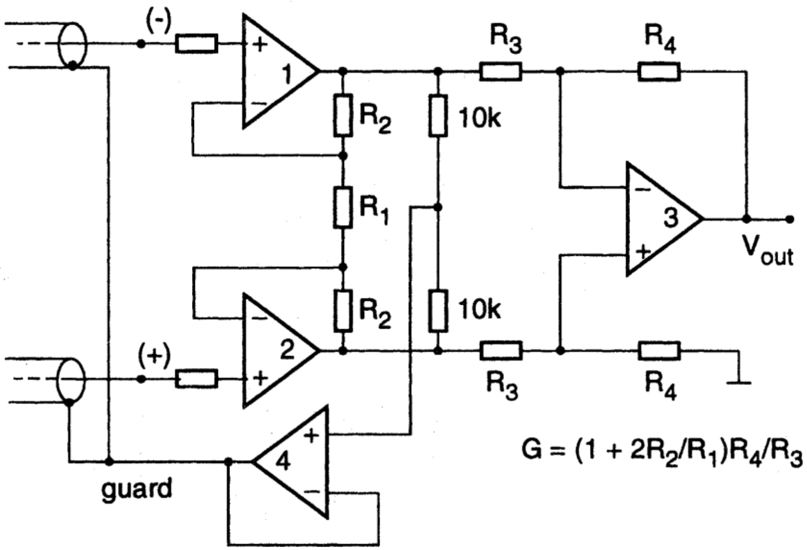
- **Additional considerations in a bioamplifier**
  - Motion artifacts
    - The contact between the electrode and the tissue changes during the relative motions between the electrodes and the tissue.
  - How to decrease the motion artifacts?
    - High input resistance of the amplifier
    - High quality electrodes
    - Reduction of the source impedance by usage of electrode gel.



Operational Amplifier (Op-amp)

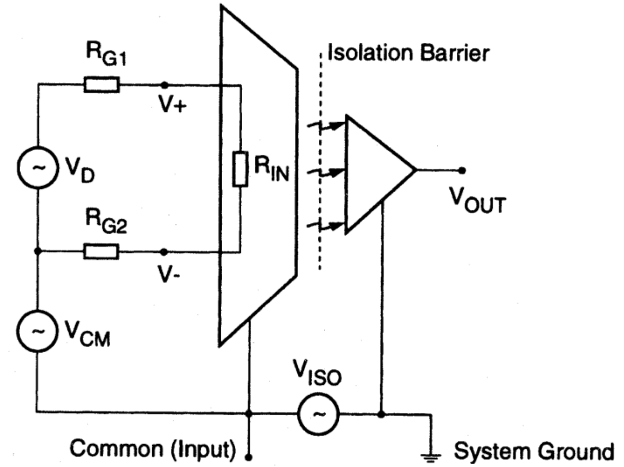
- **Additional considerations in a bioamplifier**
  - Input guarding
    - Negative feedback of common mode to reduce artifacts
    - Technique for increase both the input impedance of the amplifier of biopotentials and the CMRR
    - Instrumentation amplifier providing input guarding

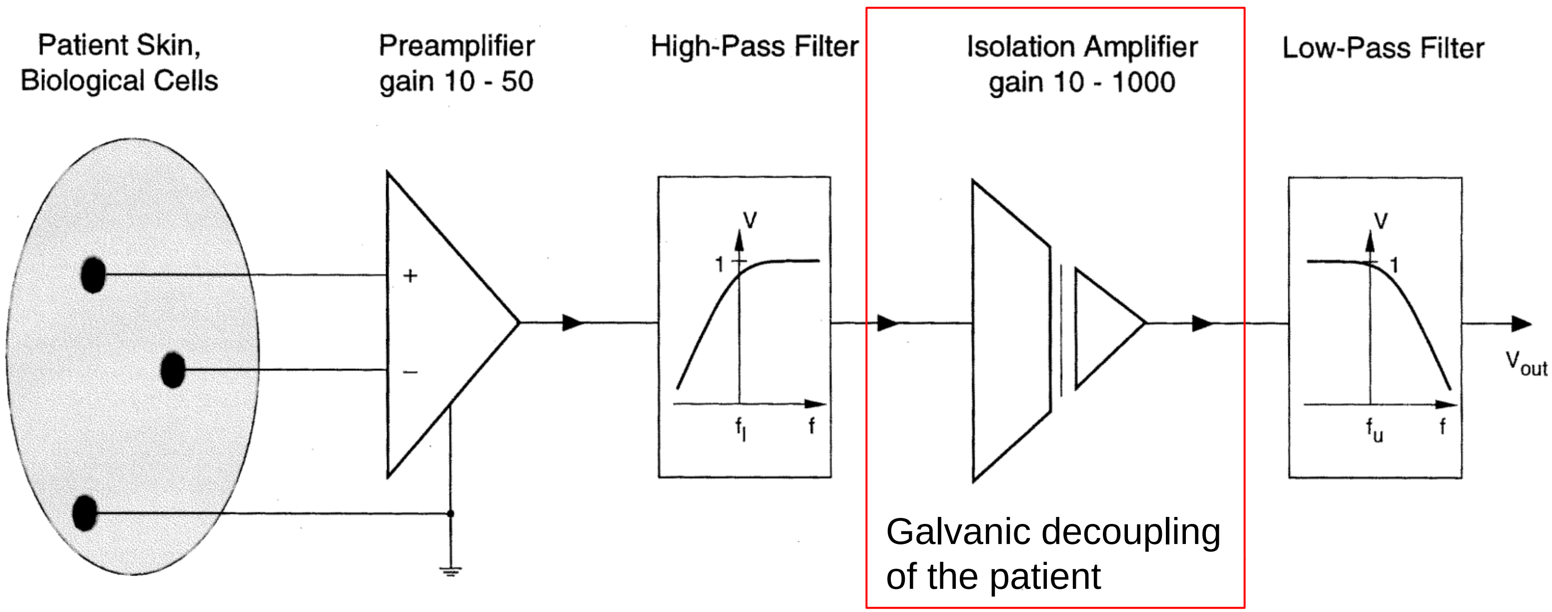




Operational Amplifier (Op-amp)

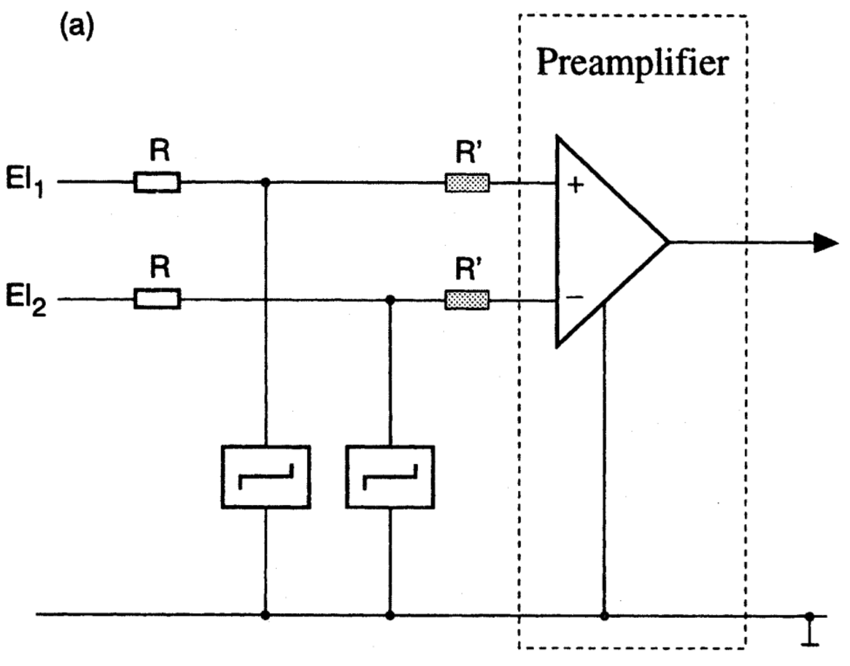
- **Additional considerations in a bioamplifier**
  - Isolation amplifier
    - Isolation is realized in the following technologies:
      - Transformer isolation
      - Opto-isolation.
    - Complete galvanic separation between the input stage (patient) and the remaining part of the measure equipment.

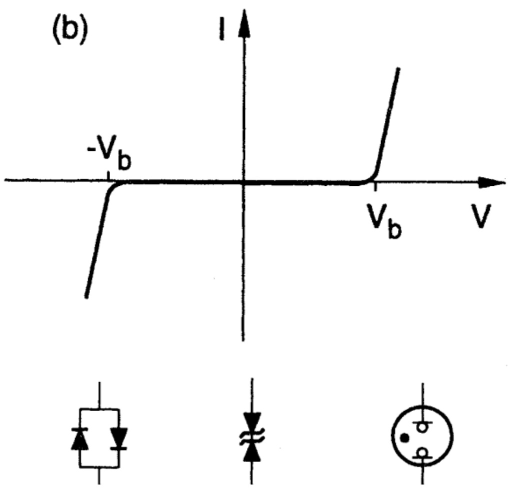




Operational Amplifier (Op-amp)

- **Additional considerations in a bioamplifier**
  - Surge protection of the bioamplifiers
    - Protection of the amplifier from damage due to surge input potentials.
    - Zener diodes and gas discharge tubes switch to conducting (short circuit) mode at high voltages (above the "breakdown" voltage

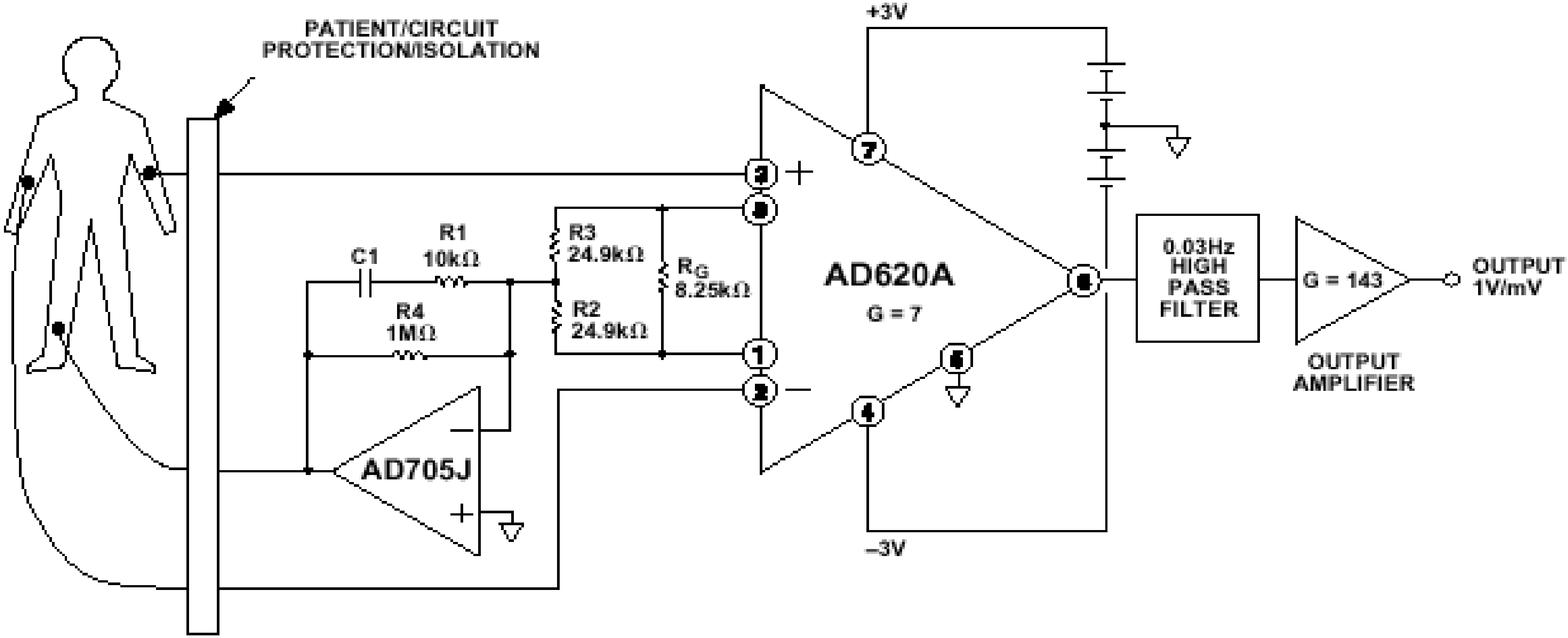




•Diodes
•Zener diodes
•Gas-discharge tubes

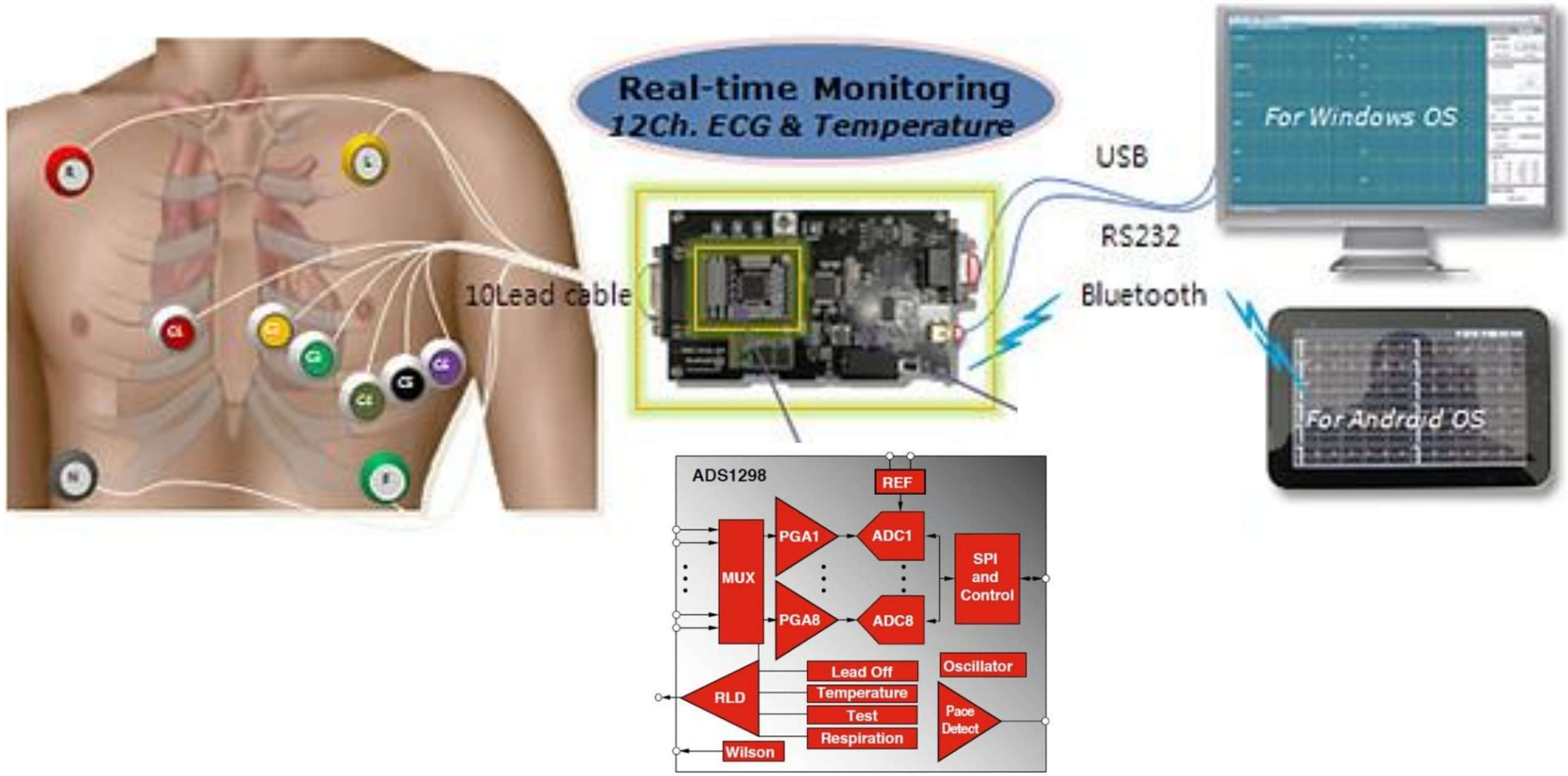
Operational Amplifier (Op-amp)

- **ECG Monitor Circuit**



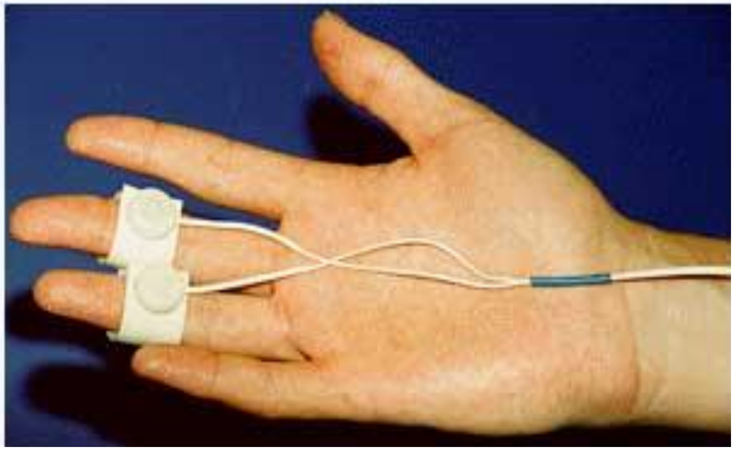
Operational Amplifier (Op-amp)

- **ECG Monitor Circuit**



Other (nonelectric) Biosignals

- **Measuring skin resistance**
  - Galvanic Skin response (GSR)
  - Electrodermal Activity (EDA)
  - Skin Conductance Level (SCL)



- **Measuring peripheral body temperature**
  - Thermistors present a resistance that changes with temperature

Other (nonelectric) Biosignals

- **Measuring blood pressure**
  - Inflate a cuff to momentarily interrupt the circulation
  - Measure the pressure at which pulsation appears again

- **Measuring blood oxygenation**
  - Measure Red/Infrared Ratio
  - Digital Signal Processing is used to do the “math”

Other (nonelectric) Biosignals

- **Measuring blood glucose**
  - Oxidase glucose on the strip
  - Measure the change (color, conductance, etc)
- **Measuring breathing capacity**
  - Flow, temperature, motion

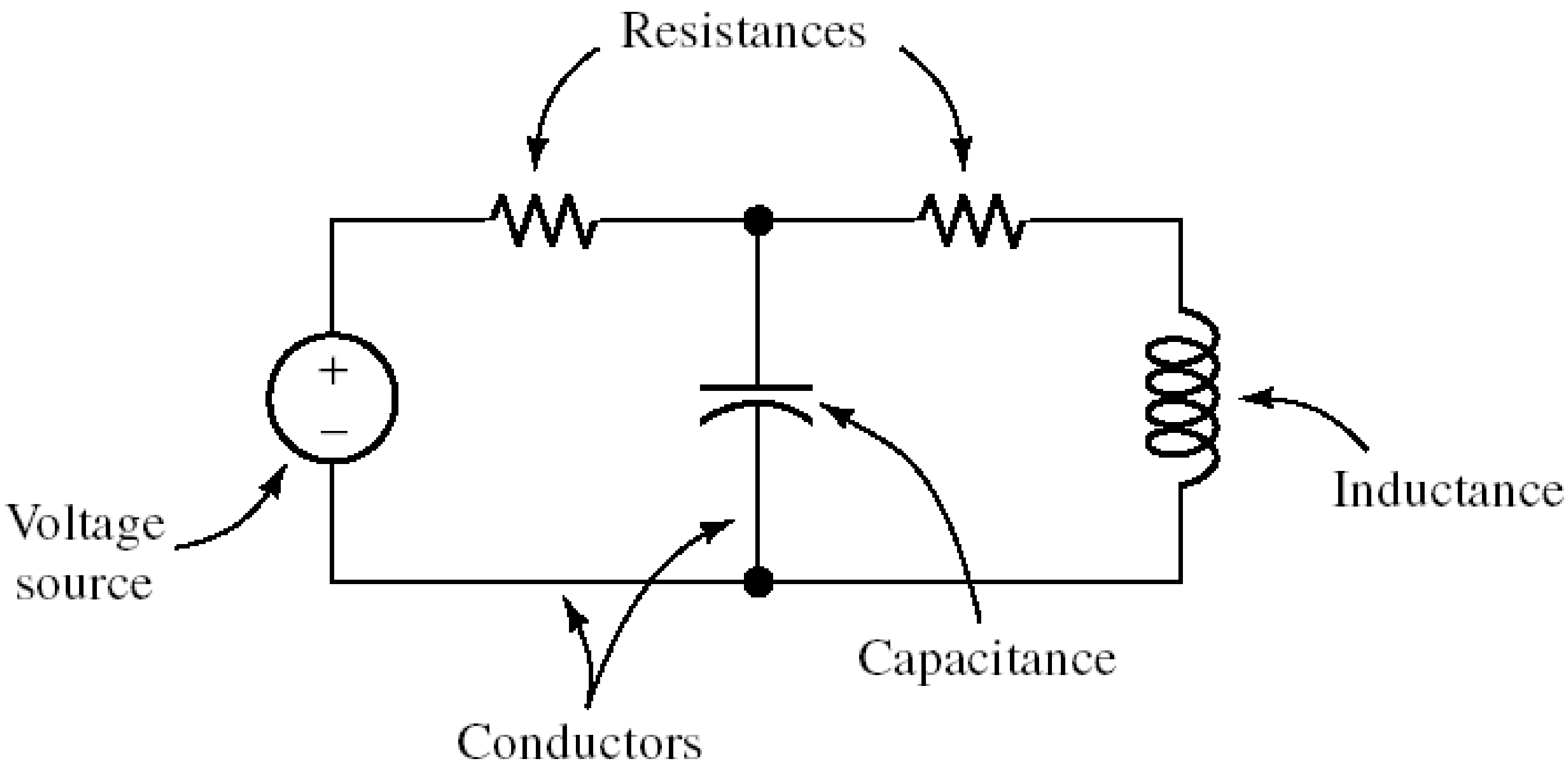
Other (nonelectric) Biosignals

- **And just about everything else**

Appendix

DC Circuits
Capacitors and Inductors
AC Circuits
Operational Amplifiers

Circuit Elements

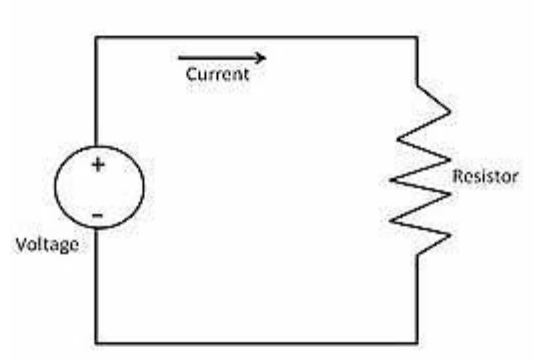


An electrical circuit consists of circuit elements such as voltage sources, resistances, inductances and capacitances that are connected in closed paths by conductors

DC Circuits

- **Current:**
  - The rate of motion of charge in a circuit
  - Symbol: I (or sometimes i).
  - SI units: C/s = ampere (A)
  - “Conventional current”
    - Assumed to consist of the motion of positive charges.
    - Conventional current flows from higher to lower potential.
  - AC/DC
    - Direct current (DC) flows in one direction around the circuit
    - Alternating current (AC) “sloshes” back and forth (time-varying that changes its sign periodically)

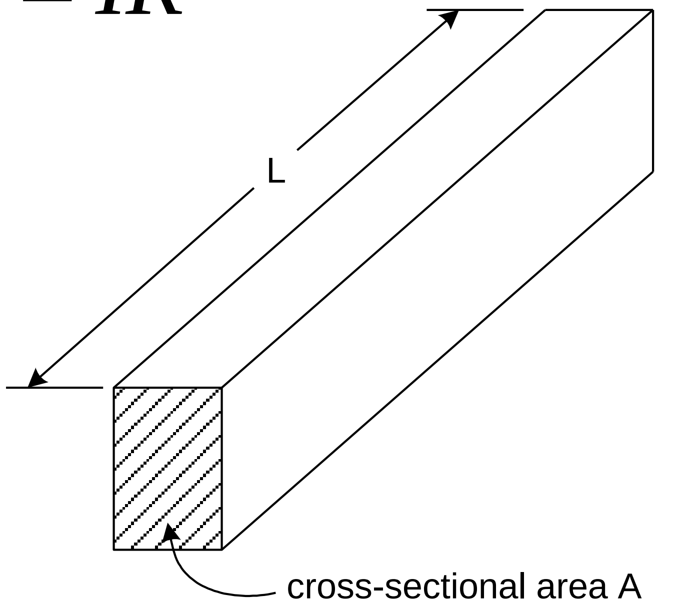
$$I = \frac{q}{t} = \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$$



DC Circuits

- **Ohm's Law and Resistance:**
  - The current that flows through an object is directly proportional to the voltage applied across the object
  - The constant of proportionality, R, is called the resistance of the object.
  - SI unit of resistance: ohm (Ω)
  - Resistance depends on the geometry of the object, and a property, resistivity, of the material from which it is made
  - Resistivity symbol: ρ
  - SI units of resistivity: ohm m (Ω m)

$$V = IR$$



$$R = \rho \frac{L}{A}$$

DC Circuits

- **Electrical Power**
  - Power is the time rate of doing work
  - Voltage is the work done per unit charge
  - Current is the time rate at which charge goes by
  - Combining
  - Ohm's Law substitutions allow us to write several equivalent expressions for power
  - Regardless of how specified, power always has SI units of watts (W)

$$P = \frac{W}{t}$$

$$V = \frac{W}{q}$$

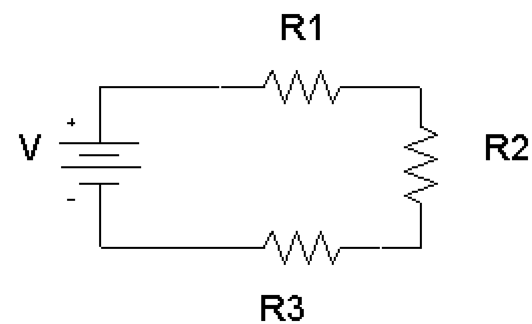
$$I = \frac{q}{t}$$

$$P = \frac{W}{t} = \frac{W}{q} \cdot \frac{q}{t} = VI$$

$$P = VI = I^2 R = \frac{V^2}{R}$$

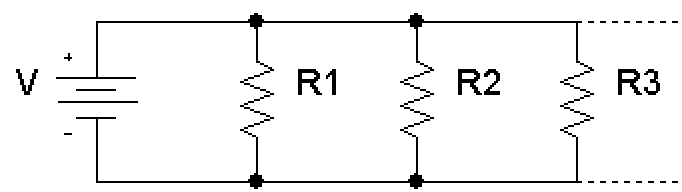
DC Circuits

- **Series Connection**
  - A circuit, or a set of circuit elements, are said to be connected “in series” if there is only one electrical path through them.
  - The same current flows through all series-connected elements. (Equation of continuity)



$$R_{eq} = R_1 + R_2 + \ldots + R_n$$

- **Parallel Connection**
  - A circuit, or a set of circuit elements, are said to be connected “in parallel” if the circuit current is divided among them.
  - The same potential difference exists across all parallel-connected elements.

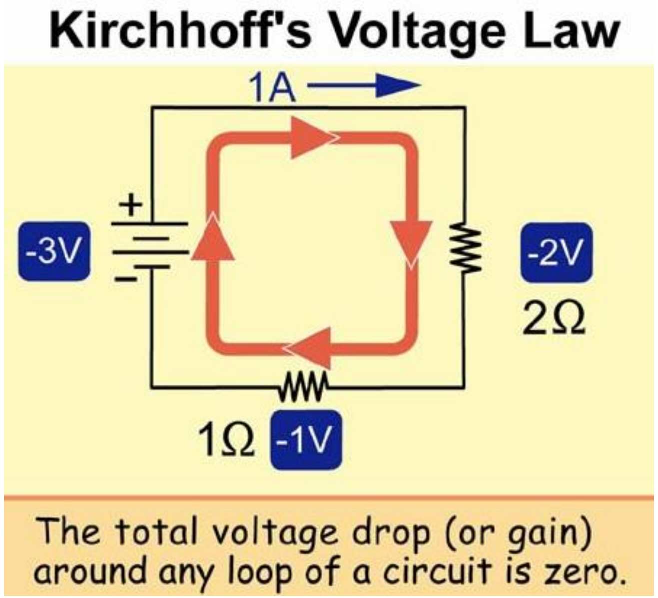


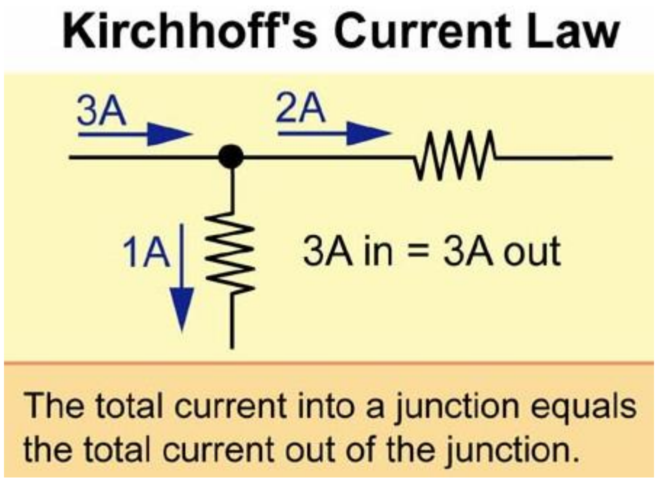
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

DC Circuits

Kirchoff's Laws

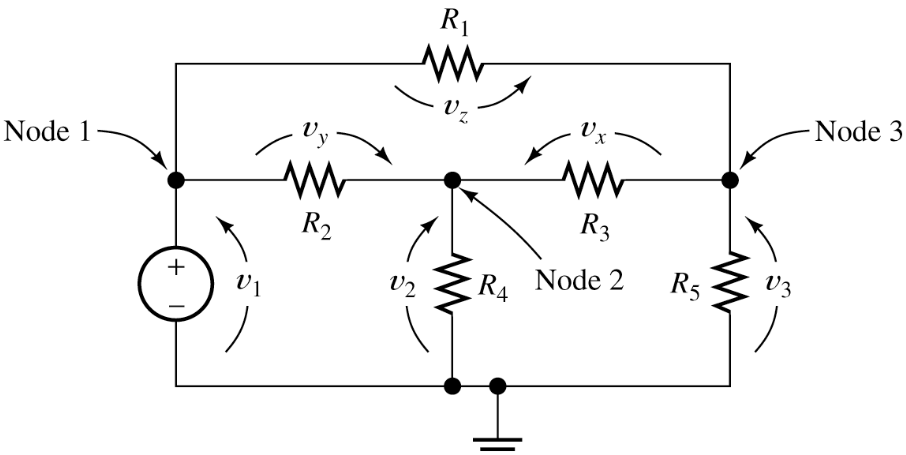
- **The Voltage Law:**
  - Around any closed loop in a circuit, the sum of the potential changes must equal zero.
  - (Energy conservation)
- **The Current Law:**
  - At any point in a circuit, the total of the currents flowing into that point must be equal to the total of the currents flowing out of that point.
  - (Charge conservation; equation of continuity)





DC Circuits

- **Node Voltage Analysis**



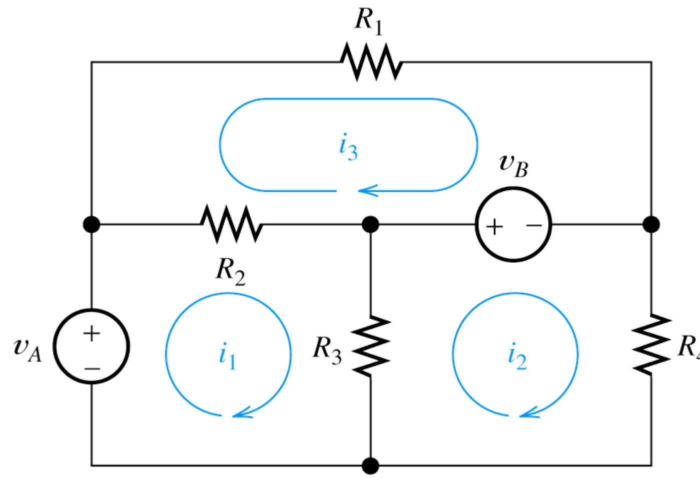
$$v_1 = v_s$$

From Kirchoff's Current Law

$$\frac{v_2 - v_1}{R_2} + \frac{v_2}{R_4} + \frac{v_2 - v_3}{R_3} = 0$$

$$\frac{v_3 - v_1}{R_1} + \frac{v_3}{R_5} + \frac{v_3 - v_2}{R_3} = 0$$

- **Mesh Current Analysis**



From Kirchoff's Voltage Law

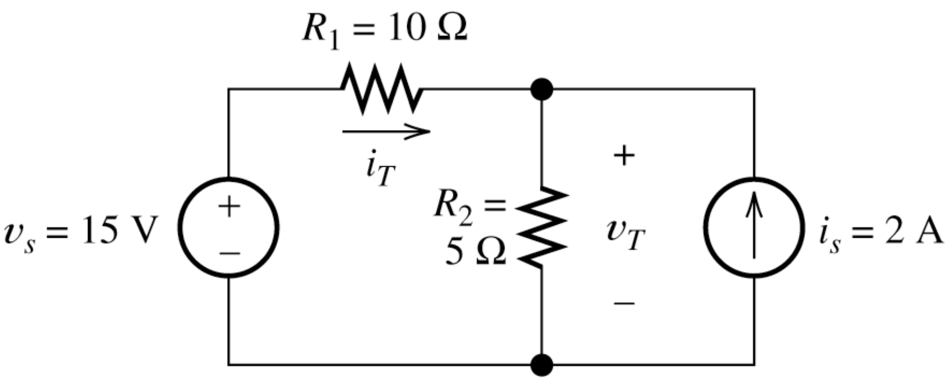
$$(i_1 - i_3)R_2 + (i_1 - i_2)R_3 - v_A = 0$$

$$(i_1 - i_2)R_3 - v_B - i_2 R_4 = 0$$

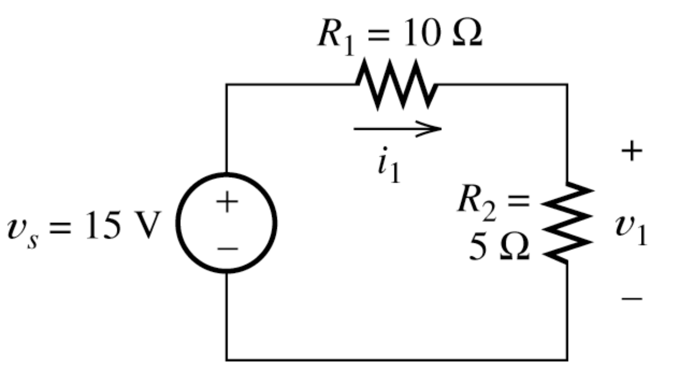
$$(i_1 - i_3)R_2 - i_3 R_1 + v_B = 0$$

DC Circuits

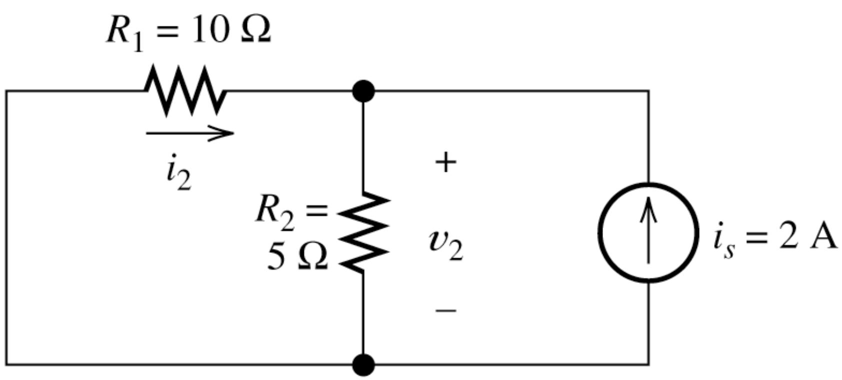
- **Superposition Principle**
  - The superposition principle states that the total response is the sum of the responses to each of the independent sources acting individually



(a) Original circuit



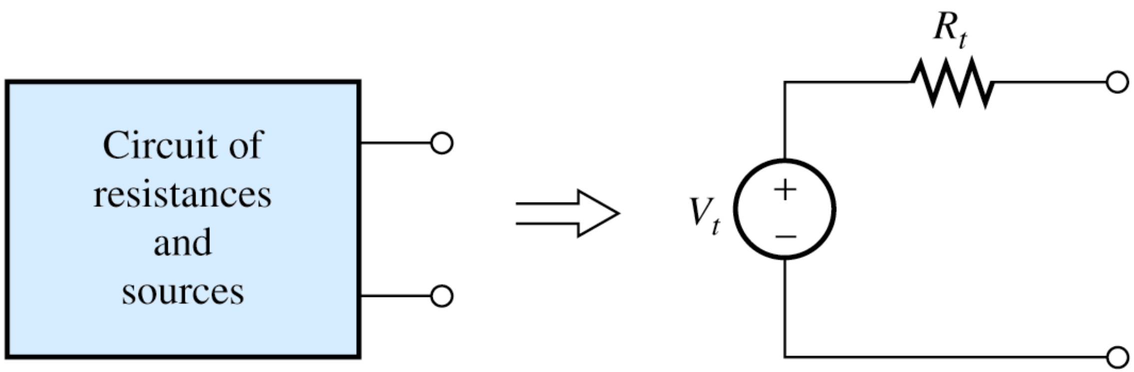
(b) Circuit with only the voltage source active

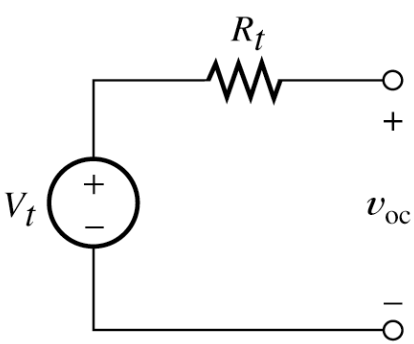


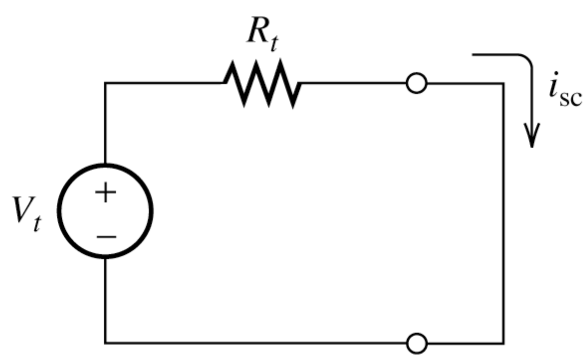
(c) Circuit with only the current source active

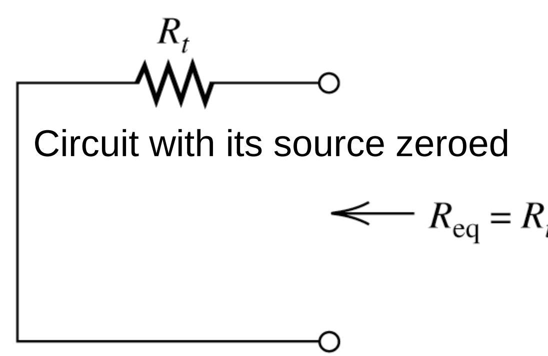
DC Circuits

- **Thévenin Equivalent Circuits**









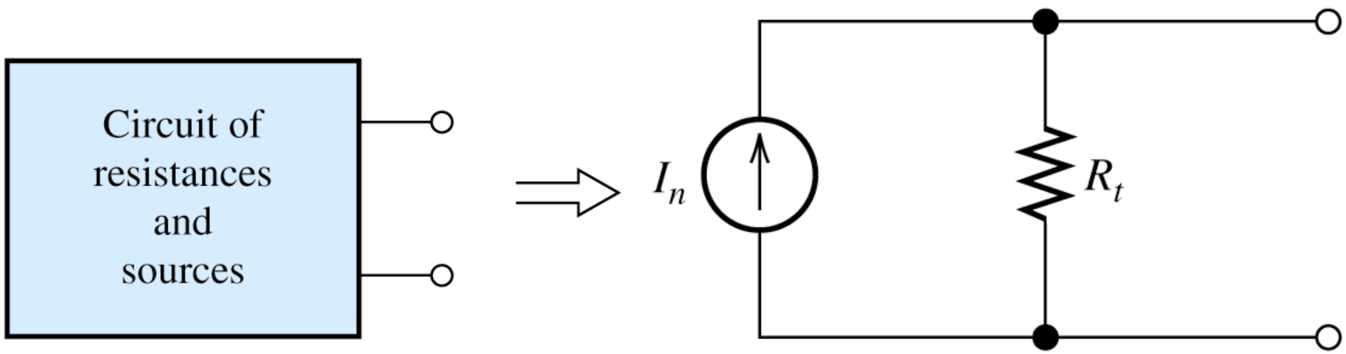
$$V_t = v_{oc}$$

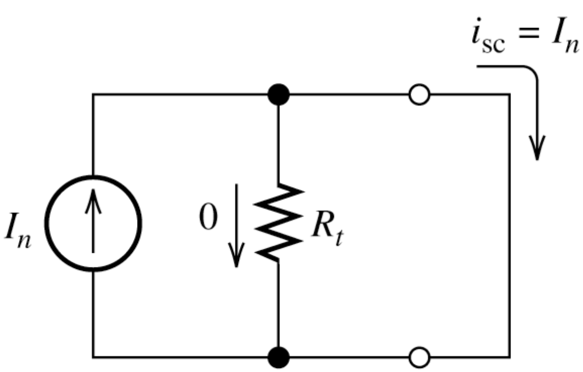
$$R_t = \frac{v_{oc}}{i_{sc}}$$

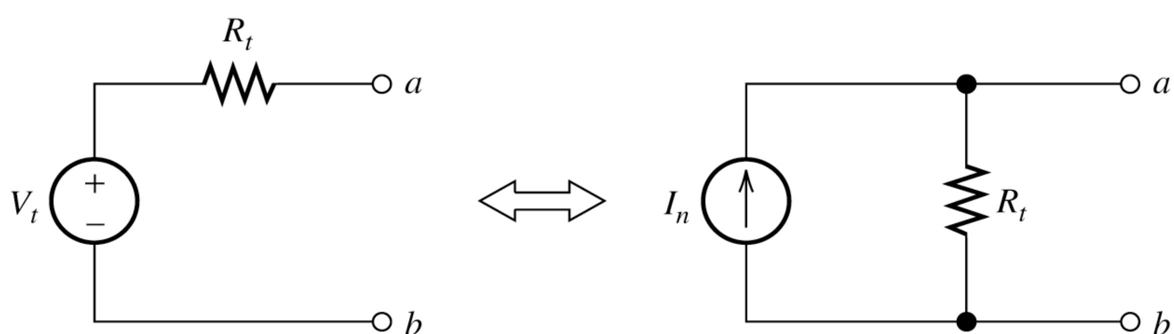
$$R_t = R_{eq}$$

DC Circuits

- **Norton Equivalent Circuits**





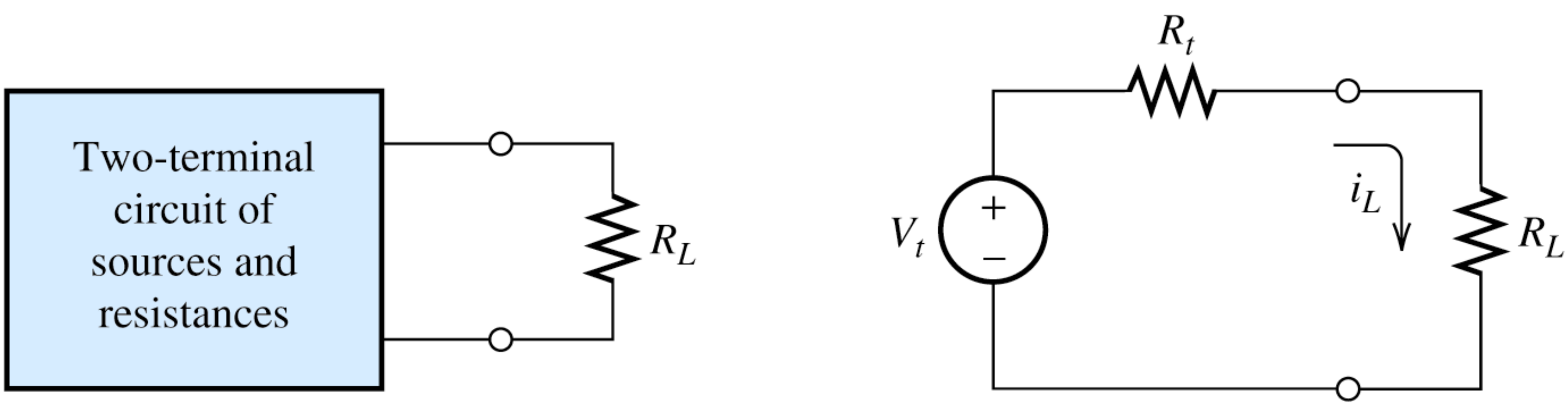


$$I_n = i_{sc}$$

$$I_n = \frac{V_t}{R_t}$$

DC Circuits

- **Maximum Power Transfer**



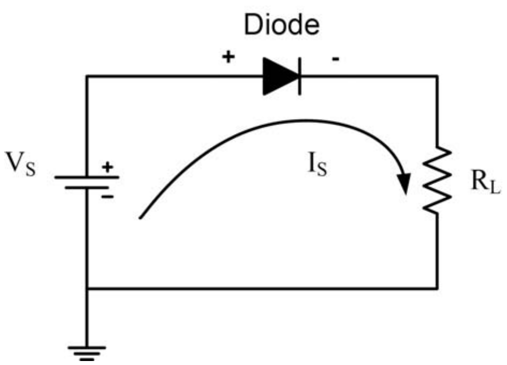
(a) Original circuit with load

(b) Thévenin equivalent circuit with load

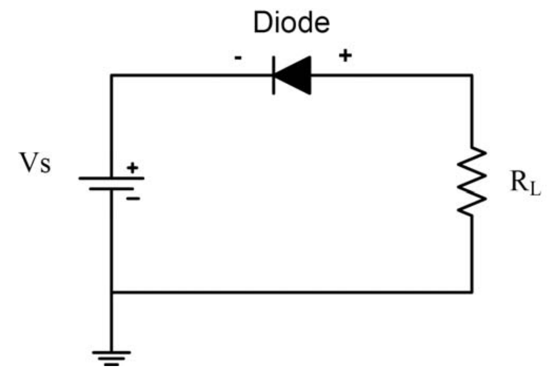
$$I_L = \frac{V_t}{R_t + R_L} \qquad P_L = i_L^2 R_L = \left(\frac{V_t}{R_t + R_L} \right)^2 R_L \qquad \frac{dP_L}{dR_L} = 0 \rightarrow R_L = R_t$$

DC Circuits

- **Diode**
  - Semiconductor device
  - Positive and negative polarities

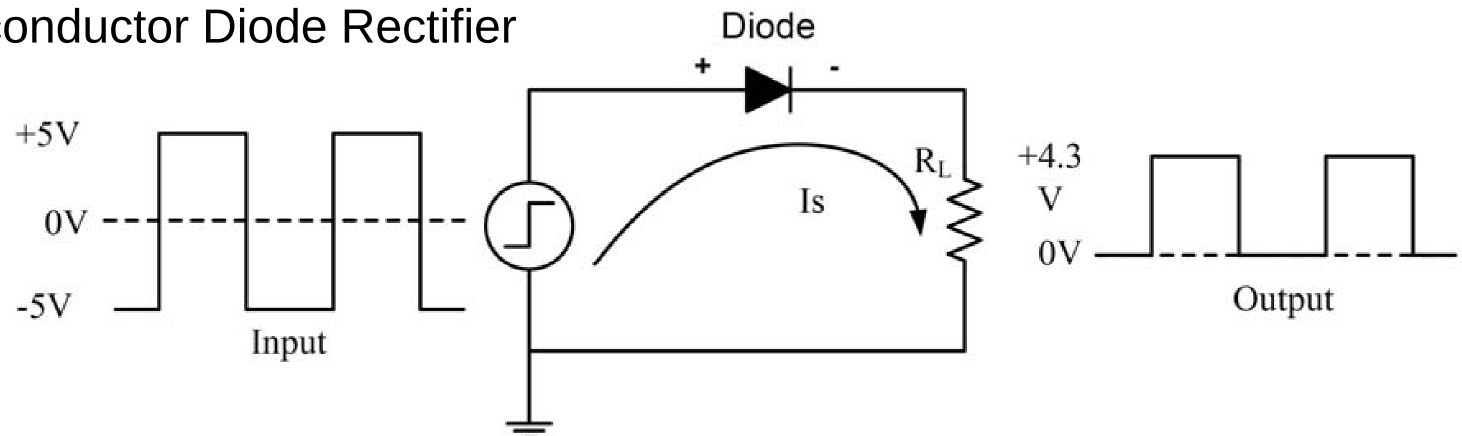


A forward-biased diode circuit



A reverse-biased diode circuit

Semiconductor Diode Rectifier



A bipolar square wave applied to a forward-biased diode circuit

Summary Table of DC Concepts

Device/Measurement	Action	Result
Series resistors	Simply add them together	R_T is larger than any single resistor value
Parallel resistors	Add them after inverting each value of the resistor, then invert the result	R_T is less than any single resistor value
Voltage drop in a series resistive circuit	The sum of drops equals the total source voltage	As the resistor value goes up, the value of the voltage drop also increases making them directly proportional
Currents in parallel circuits	Sum of branch currents equals the total current	As the resistor value goes up, the current value goes down making them inversely proportional
Capacitor	Voltage charges in a capacitor	Time constant $= R \times C$
Inductor	Current builds up in an inductor	Time constant $= \frac{L}{R}$

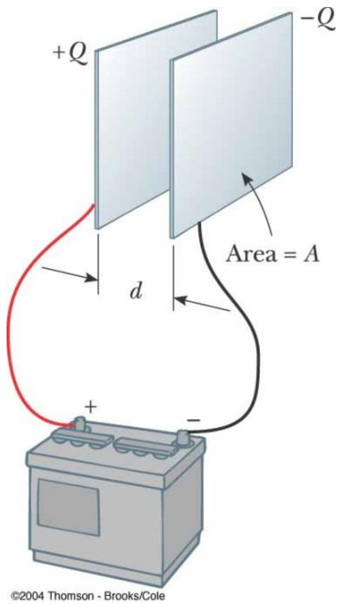
Capacitors and Inductors

- **Capacitance**
  - Ability to store charge
  - The SI unit of capacitance is the farad:
    - 1 farad = 1 F = 1 Coulomb/Volt
  - For a given charge, a capacitor with a larger capacitance will have a greater potential difference

$$Q = \frac{\varepsilon_0 A}{d}\Delta V = C\Delta V$$

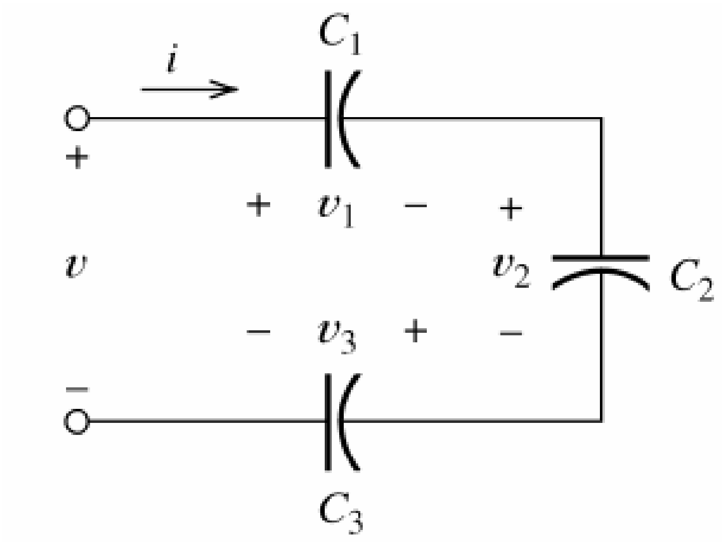
$$C = \frac{\varepsilon_0 A}{d}$$

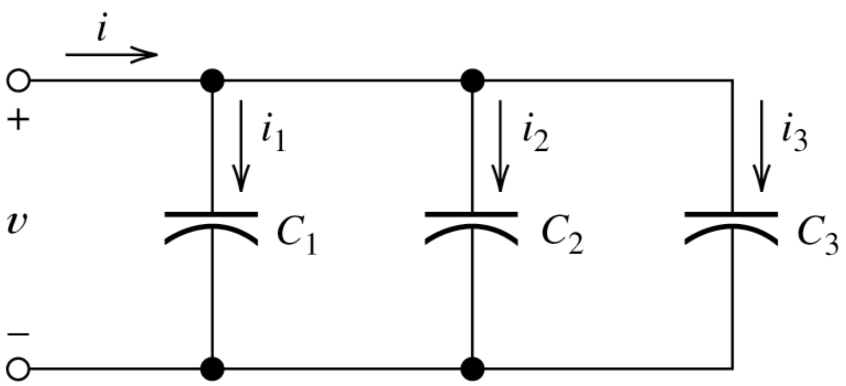
$$C = \frac{k\varepsilon_0 A}{d}$$ (with dielectric)



Capacitors and Inductors

- Like resistors, capacitors in circuits can be connected in series, in parallel, or in more-complex networks containing both series and parallel connections.



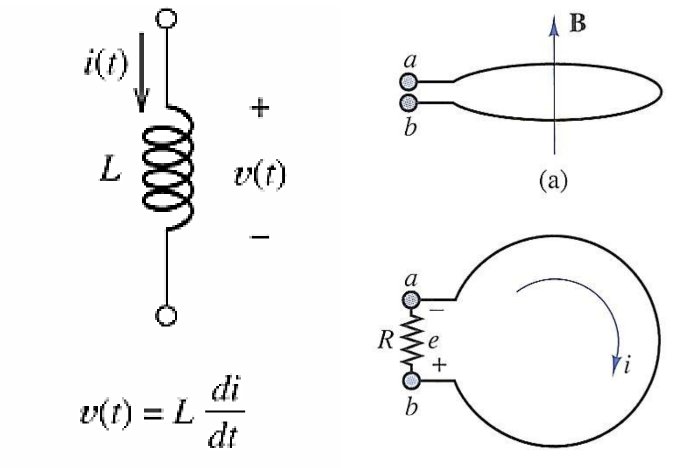


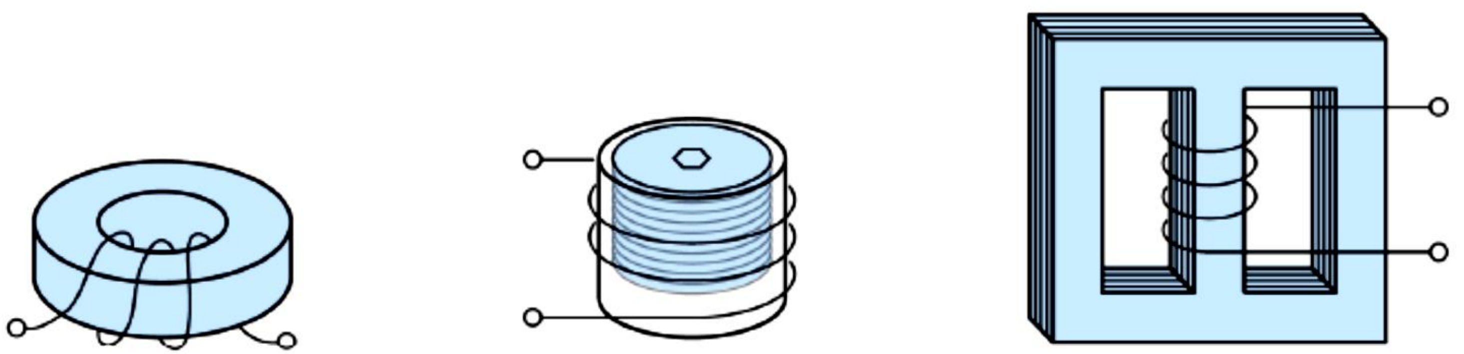
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + .. + \frac{1}{C_n}$$

$$C_{eq} = C_1 + C_2 + ... + C_n$$

Capacitors and Inductors

- **Inductance**
  - Ability to store magnetic energy
  - The polarity of the voltage is such as to oppose the change in current (Lenz's law).
  - Inductance, unit: henry [H]





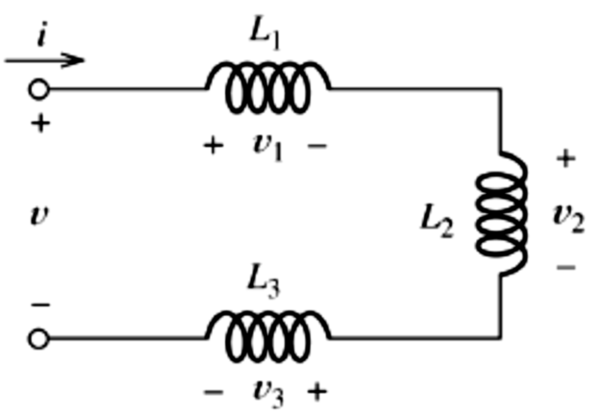
(a) Toriodal inductor

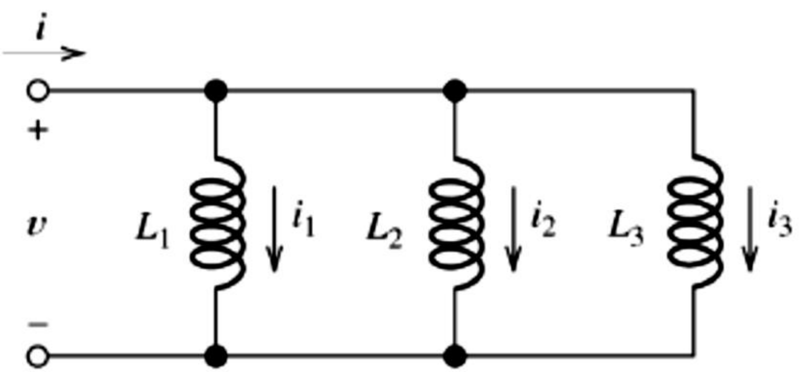
(b) Coil with an iron-oxide slug that can be screwed in or out to adjust the inductance

(c) Inductor with a laminated iron core

Capacitors and Inductors

- **Like resistors, inductors in circuits can be connected in series, in parallel, or in more-complex networks containing both series and parallel connections.**



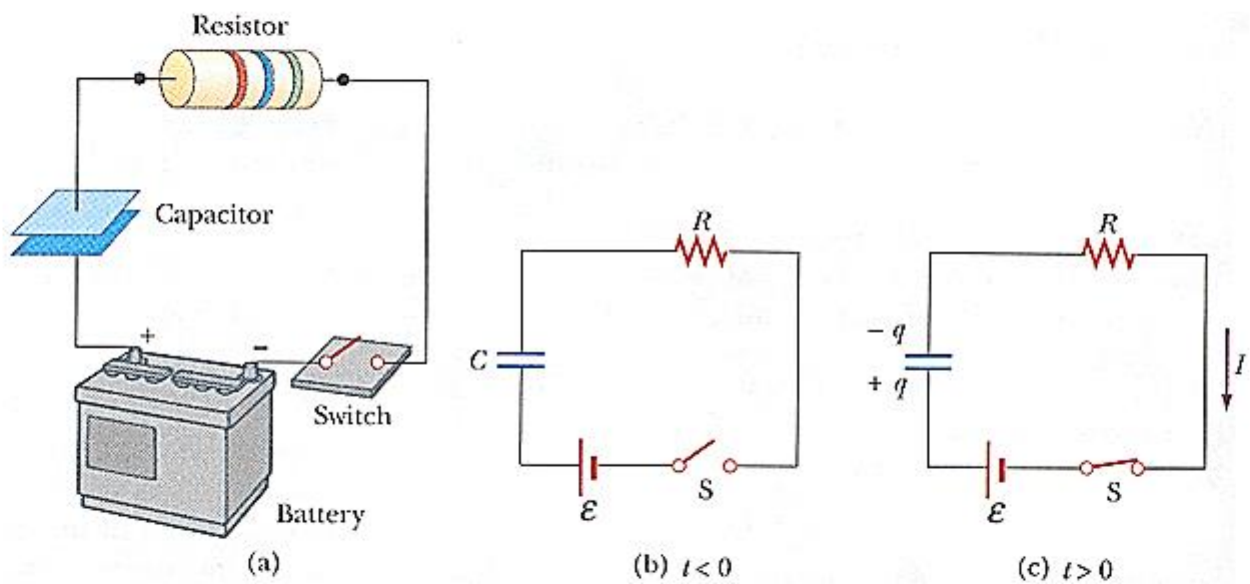


$$L_{eq} = L_1 + L_2 + L_3$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1 + L_2 + L_3}$$

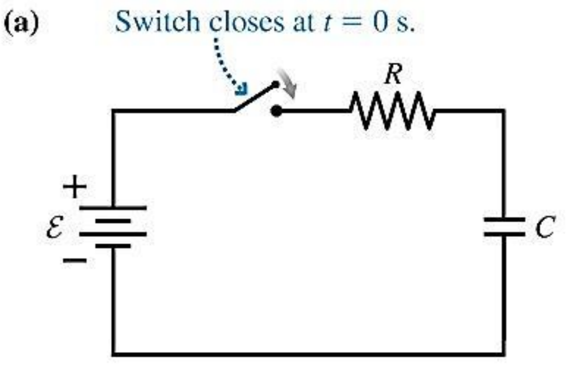
RC Circuits

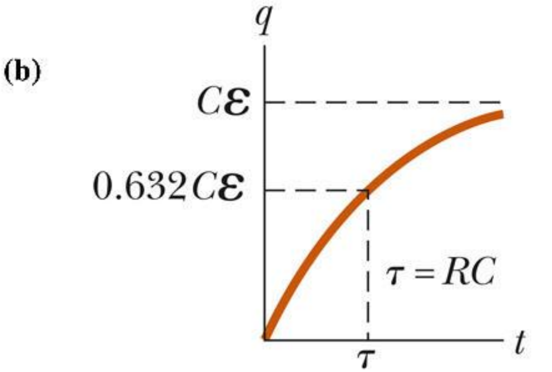
- **A capacitor connected in series with a resistor is part of an RC circuit.**
  - Resistance limits charging current
  - Capacitance determines ultimate charge
- **Unlike a battery, a capacitor cannot provide a constant source of potential difference.**
  - This value is constantly changing as the charge leaves the plate.
  - Current due to a discharging capacitor is finite and changes over time.



RC Circuits

- **Charging the capacitor**
  - Assume no initial charge in the capacitor
  - At the instant the source is connected, the capacitor starts to charge.
    - The capacitor continues to charge until it reaches its maximum charge (Q = Cε)
  - Once the capacitor is fully charged, the current in the circuit is zero.
    - The potential difference across the capacitor matches that supplied by the battery
  - The charge on the capacitor increases exponentially with time
    - τ is the time constant
    - τ = RC





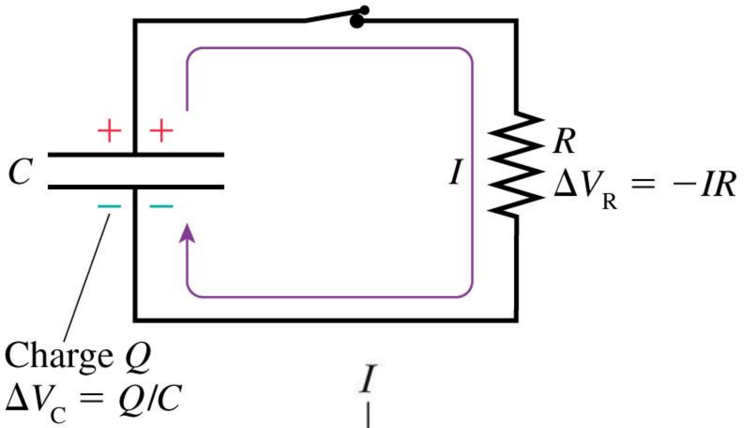
$$q(t) = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right) = Q_0\left(1 - e^{-\frac{t}{\tau}}\right)$$

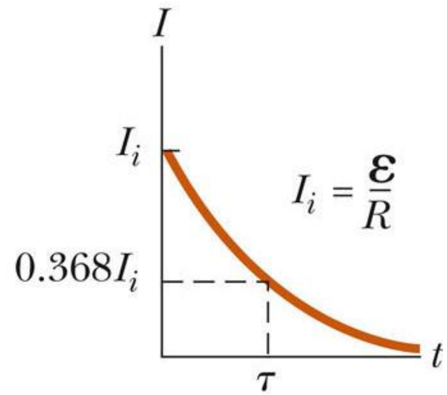
$$I(t) = \frac{dq}{dt} = \frac{C\varepsilon}{RC} e^{-\frac{t}{RC}} = I_o e^{-\frac{t}{\tau}}$$

RC Circuits

- **Discharging the capacitor**
  - Assume a fully charged capacitor
  - At the instant the switch closes, the capacitor starts to charge.
    - The capacitor continues to discharge until it reaches 0
  - Once the capacitor is fully discharged, the current in the circuit is zero.

  - The charge on the capacitor decreases exponentially with time
    - τ is the time constant
    - τ = RC



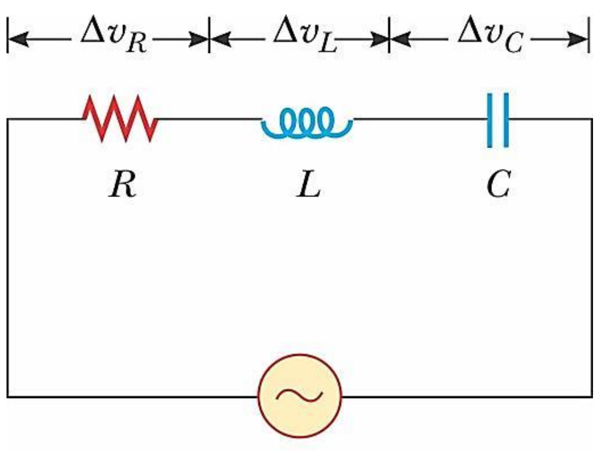


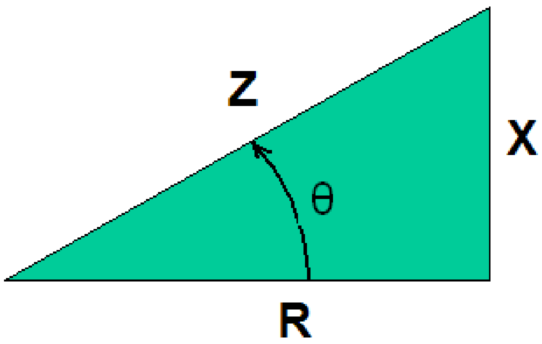
$$q(t) = C\varepsilon e^{-\frac{t}{RC}} = Q_0 e^{-\frac{t}{\tau}}$$

$$I(t) = \frac{dQ}{dt} = \frac{C\varepsilon}{RC} e^{-\frac{t}{\tau}} = I_0 e^{-\frac{t}{\tau}}$$

AC Circuits

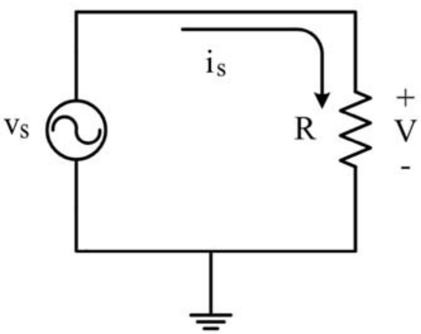
- **Instantaneous voltage** $v(t) = V_{max} \sin \omega t$
- **Instantaneous current** $i(t) = I_{max} \sin(\omega t - \theta)$ $I_{max} = V_{max} / |Z|$
  - θ is the phase angle $\theta = \tan^{-1}(X/R)$ $Z = V / I$ $|Z| \equiv \sqrt{R^2 + (X_L - X_C)^2}$
- **In phasor form** $V = V_{rms} \angle 0$ $I = I_{rms} \angle \theta$ $V_{rms} = V_{max}/\sqrt{2}$, $I_{rms} = I_{max}/\sqrt{2}$
- **Impedance** $Z = R + jX$ $X = X_L - X_c$
  - In series $Z_{eq} = (R_1 + R_2) + j(X_1 + X_2)$
  - In parallel $1/Z_{eq} = 1/(R_1 + jX_1) + 1/(R_2 + jX_2)$



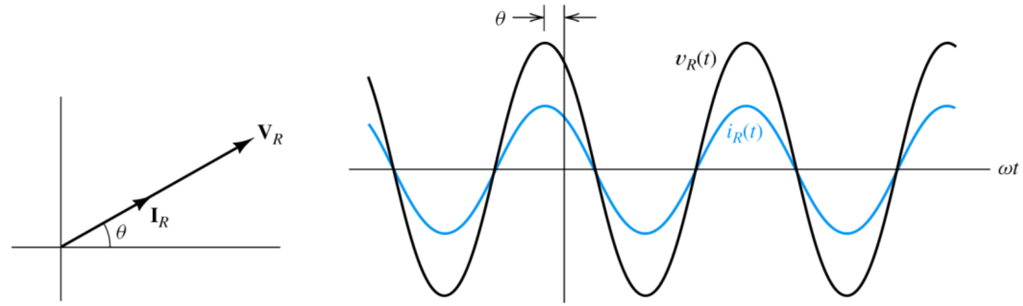


AC Circuits

Voltage and current waveforms in a resistive circuit



$$\mathbf{V}_R = R\mathbf{I}_R$$

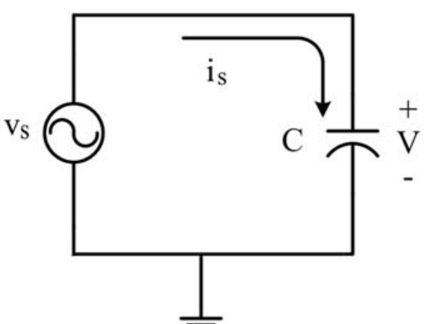


(a) Phasor diagram (b) Current and voltage versus time

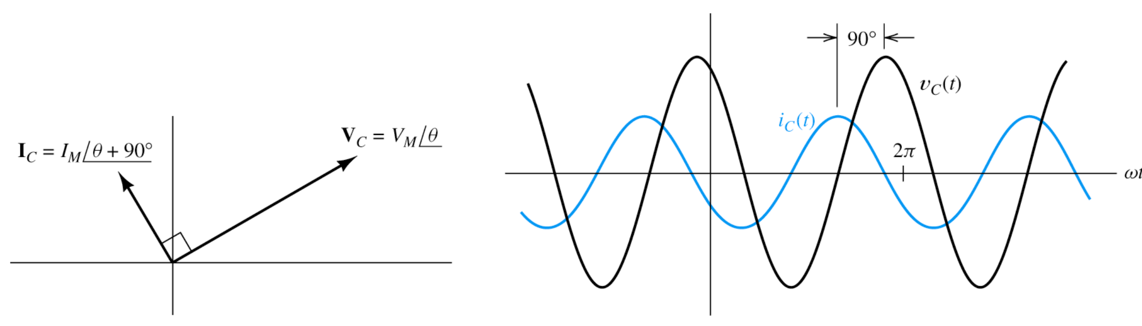
Figure 5.9 For a pure resistance, current and voltage are in phase.

$$Z_R = R$$

Voltage and current waveforms in a capacitive circuit



$$\mathbf{V}_C = Z_C\mathbf{I}_C$$



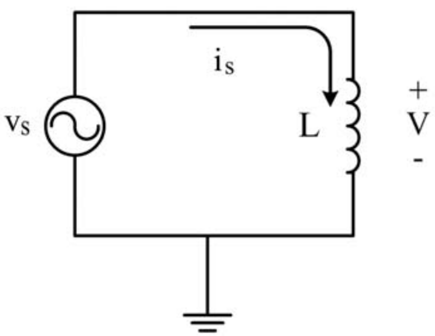
(a) Phasor diagram (b) Current and voltage versus time

Figure 5.8 Current leads voltage by 90° in a pure capacitance.

$$Z_C = -j\frac{1}{\omega C} = \frac{1}{j\omega C} = \frac{1}{\omega C}\angle -90^\circ$$

AC Circuits

Voltage and current waveforms in an inductive circuit



$$\mathbf{V}_L = Z_L \mathbf{I}_L$$

$$\mathbf{V}_L = j\omega L \times \mathbf{I}_L$$

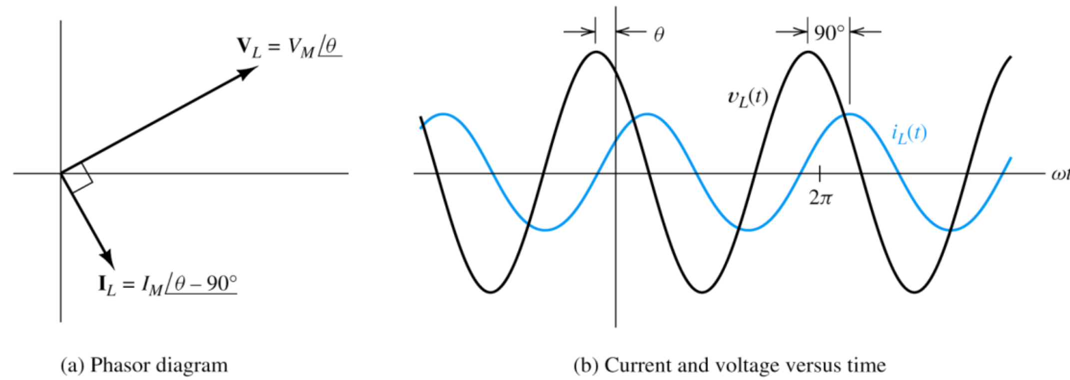
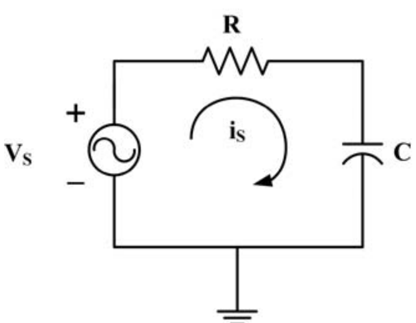
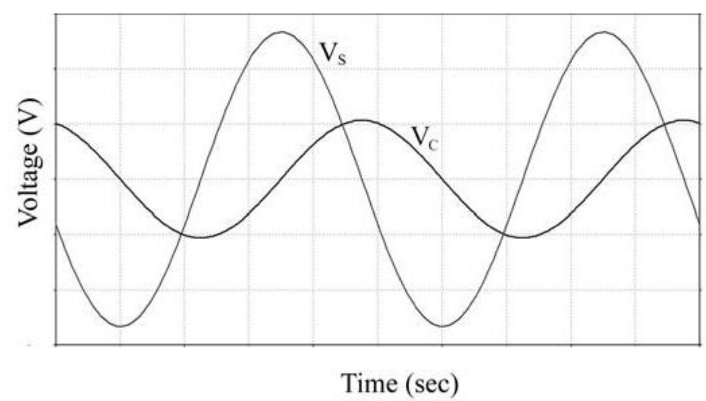


Figure 5.7 Current lags voltage by 90° in a pure inductance.

$$Z_L = j\omega L = \omega L \angle 90^\circ$$

Voltage and current waveforms in an RC series circuit

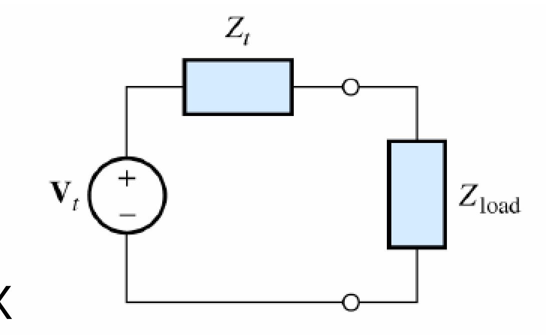




AC Circuits

- Power can be expressed in rectangle form

$S = P + jQ$



- P- real power

$P = V_{rms} I_{rms} \cos(\theta) = I^2_{rms} R$

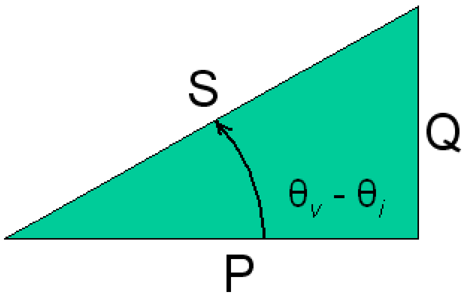
- Q–reactive power

$Q = V_{rms} I_{rms} \sin(\theta) = V^2_{rms}/X$

$S^2 = P^2 + Q^2$

- Power factor

$PF = \cos(\theta) = \cos(\theta_v - \theta_i)$



- Maximum Power Transfer

$\cos(\theta) = 1 \rightarrow \theta = 0$

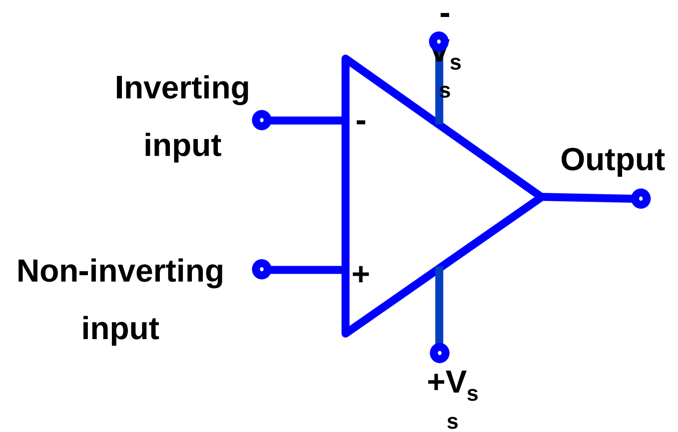
$$Z_{load} = Z_t^*$$

Summary Table of AC Concepts

Device/Measurement	Action	Result
Inductor	Inductive voltage leads the current by 90°	The inductor is open at high frequency
Capacitor	Capacitive current leads the voltage by 90°	The capacitor acts as a short at high frequency
Resistor	$R\ \Omega\angle 0°$	Real number
Capacitive reactance	$X_C\Omega\angle -90°$	$-J\ X_C\Omega$
Inductive reactance	$X_L\Omega\angle 90°$	$+J\ X_L\Omega$
Capacitive circuit	$R - J\ X_C$	$R\angle 0° + X_C\angle -90°$
Inductive circuit	$R + J\ X_L$	$R\angle 0° + X_L\angle 90°$

Operational Amplifier (Op-amp)

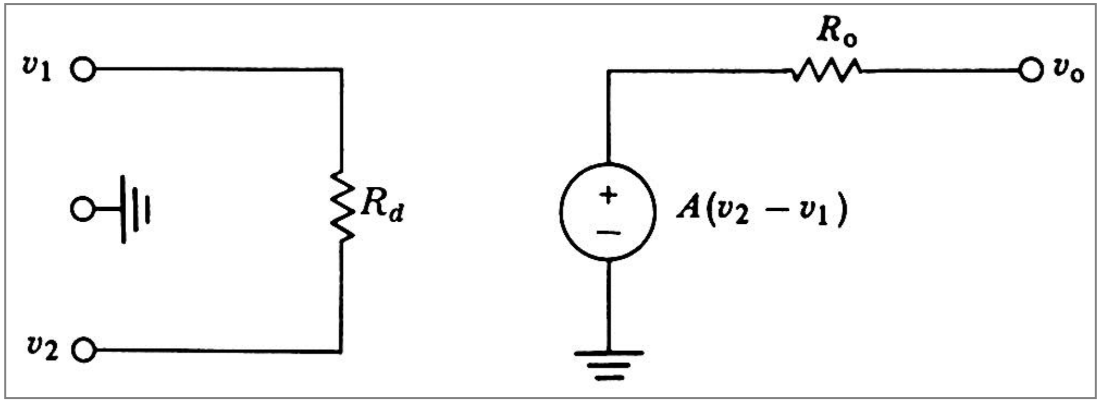
- **The op-amp is a device for increasing the power of a signal.**
  - It does this by taking power from a power supply and controlling the output to match the input signal shape but with a larger amplitude (Amplification).
- **The op-amp is used also to perform arithmetic operations (addition, subtraction, multiplication) with signals.**
- **The properties of the negative feedback loop determine the properties of the circuit containing an op-amp.**
- **It has two inputs: the inverting input (-) and the non-inverting input (+), and one output.**
- **It has usually two supplies (±Vss) but it can work with one.**



Symbol of op-amplifier

Operational Amplifier (Op-amp)

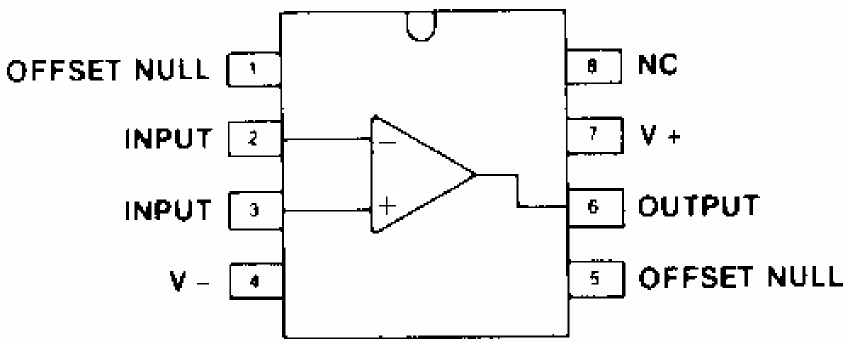
- **Ideal Op-Amp**
  - Op-amp equivalent circuit:



  - The two inputs are ʋ1 and ʋ 2.
  - A differential voltage between them causes current flow through the differential resistance Rd.
  - The differential voltage is multiplied by A (the open-loop gain of the op amp) to generate the output-voltage source
  - Any current flowing to the output terminal vo must pass through the output resistance Ro.

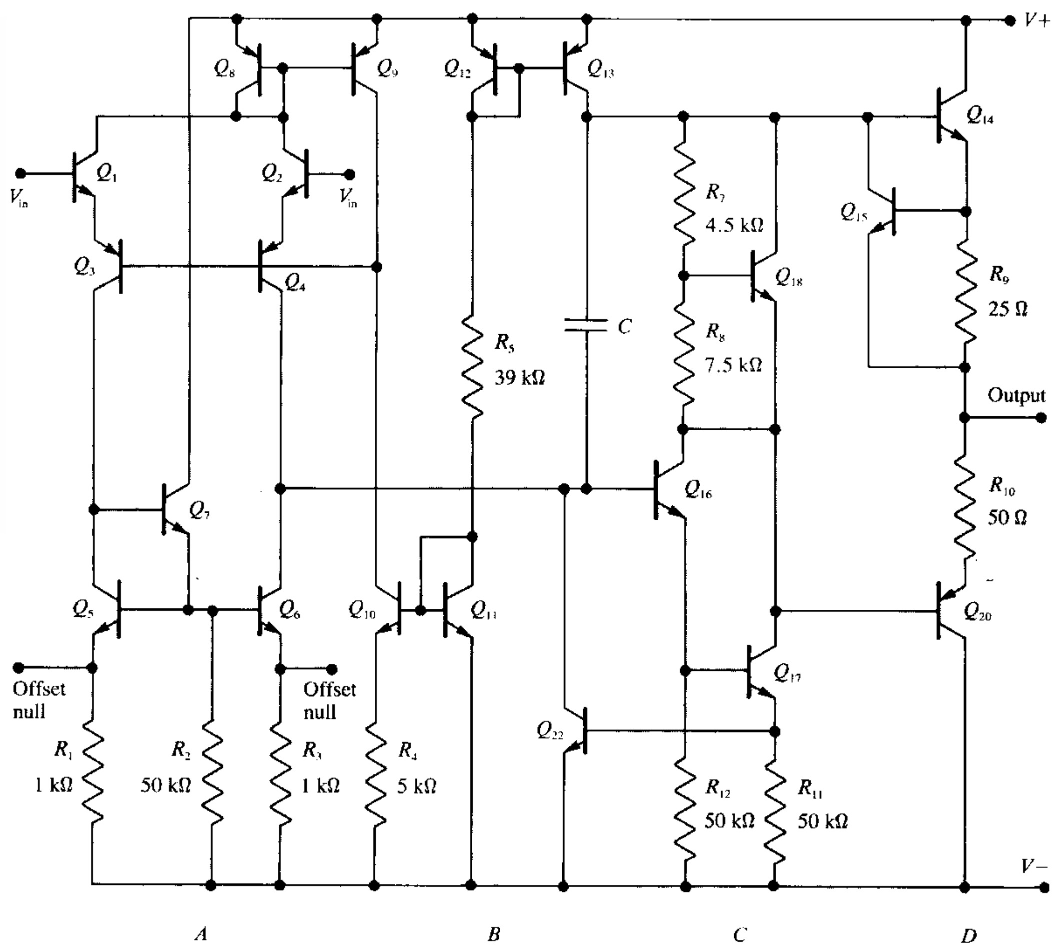
Operational Amplifier (Op-amp)

- **Inside the Op-Amp (IC-chip)**



741 op amp

20 transistors
11 resistors
1 capacitor



Operational Amplifier (Op-amp)

- **Real vs. Ideal Op-amp**

Parameter	Ideal Op Amp	Typical Op Amp
Open-loop voltage gain **A**	∞	$10^5 – 10^9$
Common mode voltage gain	0	10^{-5}
Frequency response **f**	∞	1- 20 MHz
Input impedance $\mathbf{Z_{in}}$	∞	10^6 Ω (bipolar) $10^9–10^{12}$ Ω (FET)
Output impedance $\mathbf{Z_{out}}$	0	100 – 1000 Ω

Operational Amplifier (Op-amp)

- **Summing Point Constraint**
  - In a negative feedback system, the ideal op-amp output voltage attains the value needed to force the differential input voltage and input current to zero.

- **Circuit solution**

1. Verify that negative feedback is present.
2. Assume that the differential input voltage and the input current of the op amp are forced to zero. (This is the summing-point constraint.)
3. Apply standard circuit-analysis principles, such as Kirchhoff's laws and Ohm's law, to solve for the quantities of interest.

Operational Amplifier (Op-amp)

- **Applying the Summing Point Constraint**

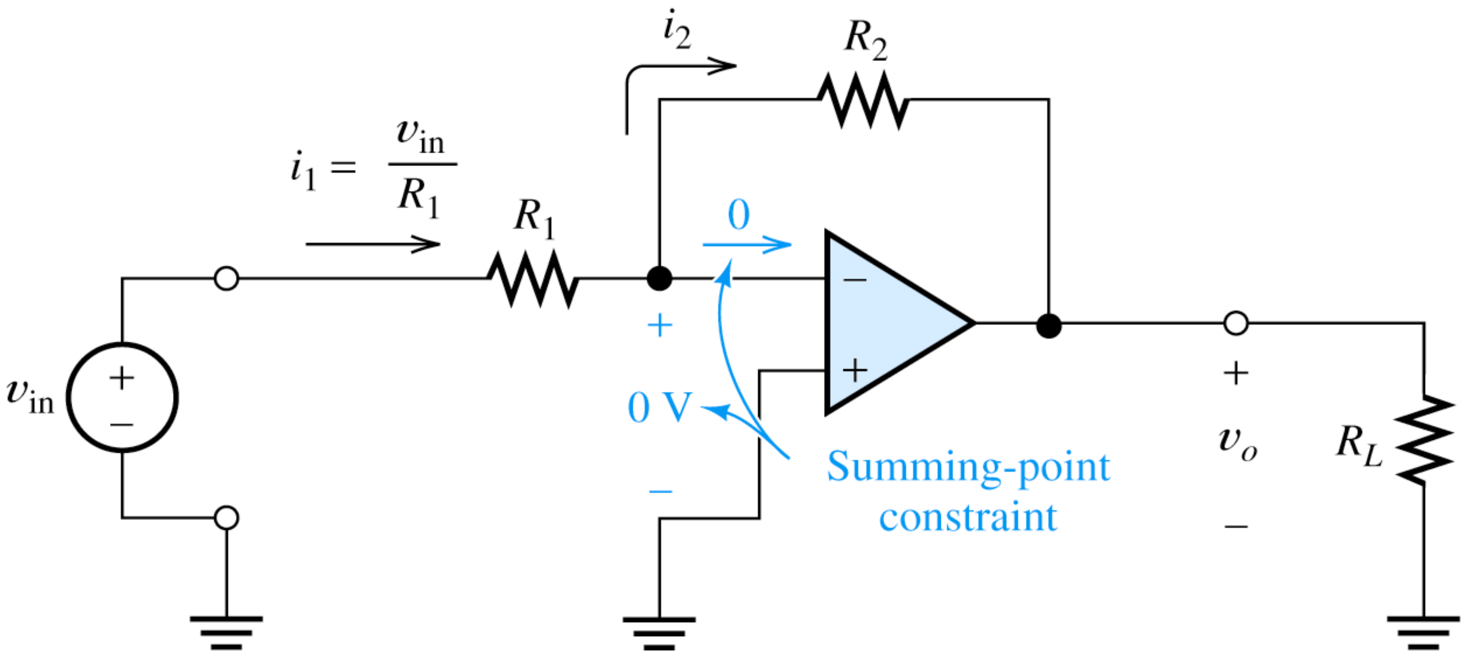
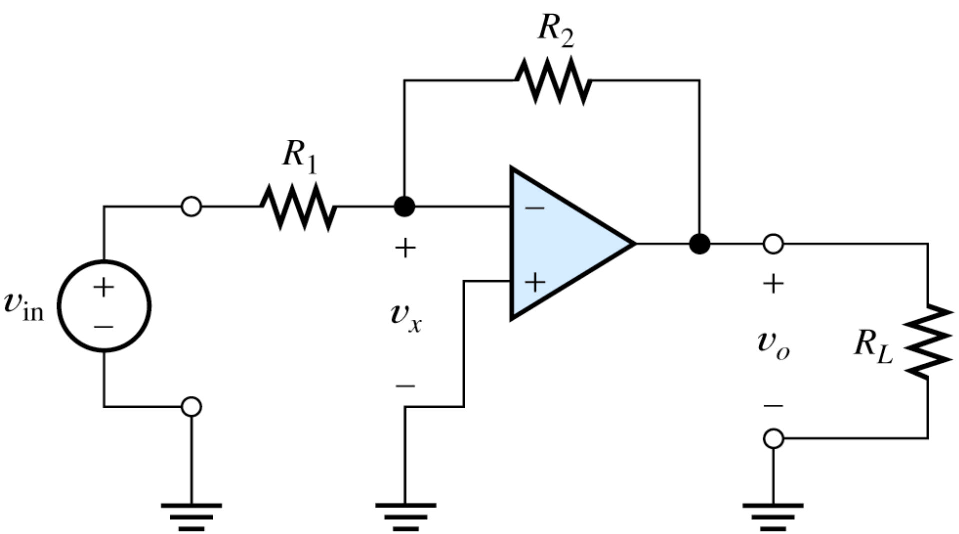


Figure 14.5 We make use of the summing-point constraint in the analysis of the inverting amplifier.

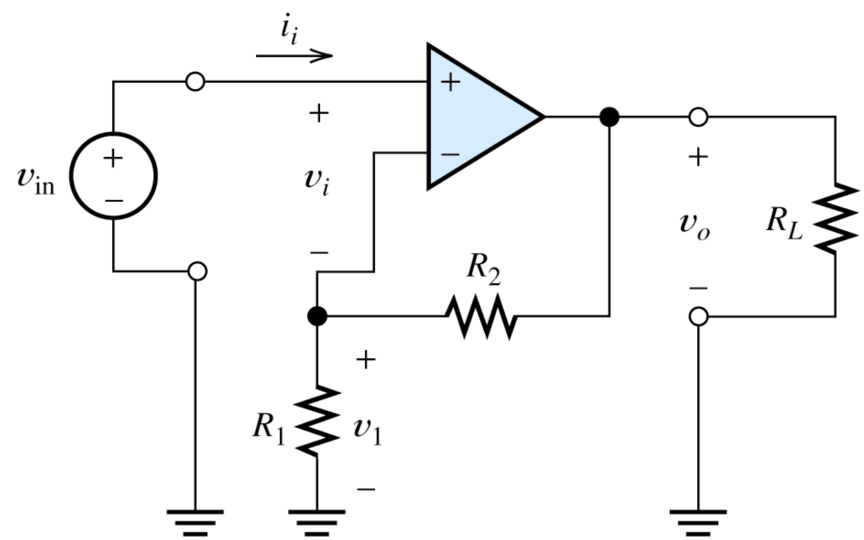
Operational Amplifier (Op-amp)

- **Inverting Amplifier**



$$A_v = \frac{v_o}{v_{in}} = -\frac{R_2}{R_1}$$

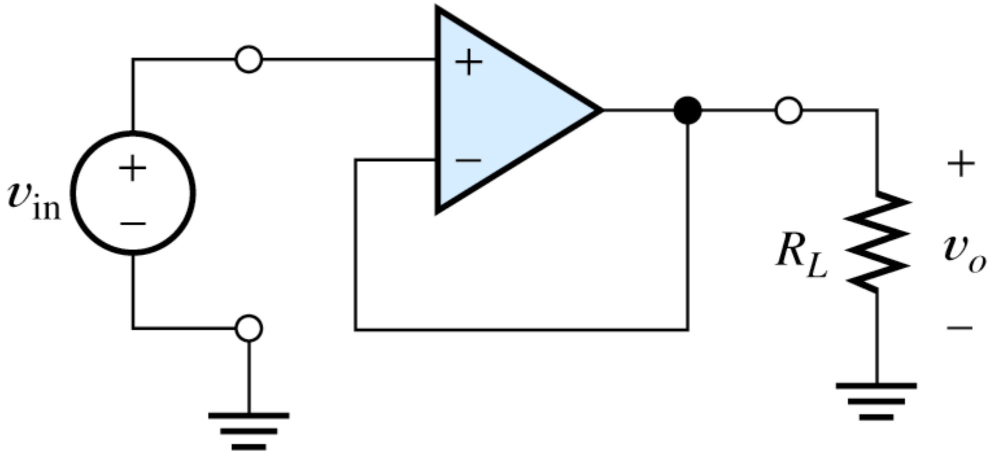
- **Non-inverting Amplifiers**



$$A_v = \frac{v_o}{v_{in}} = 1 + \frac{R_2}{R_1}$$

Operational Amplifier (Op-amp)

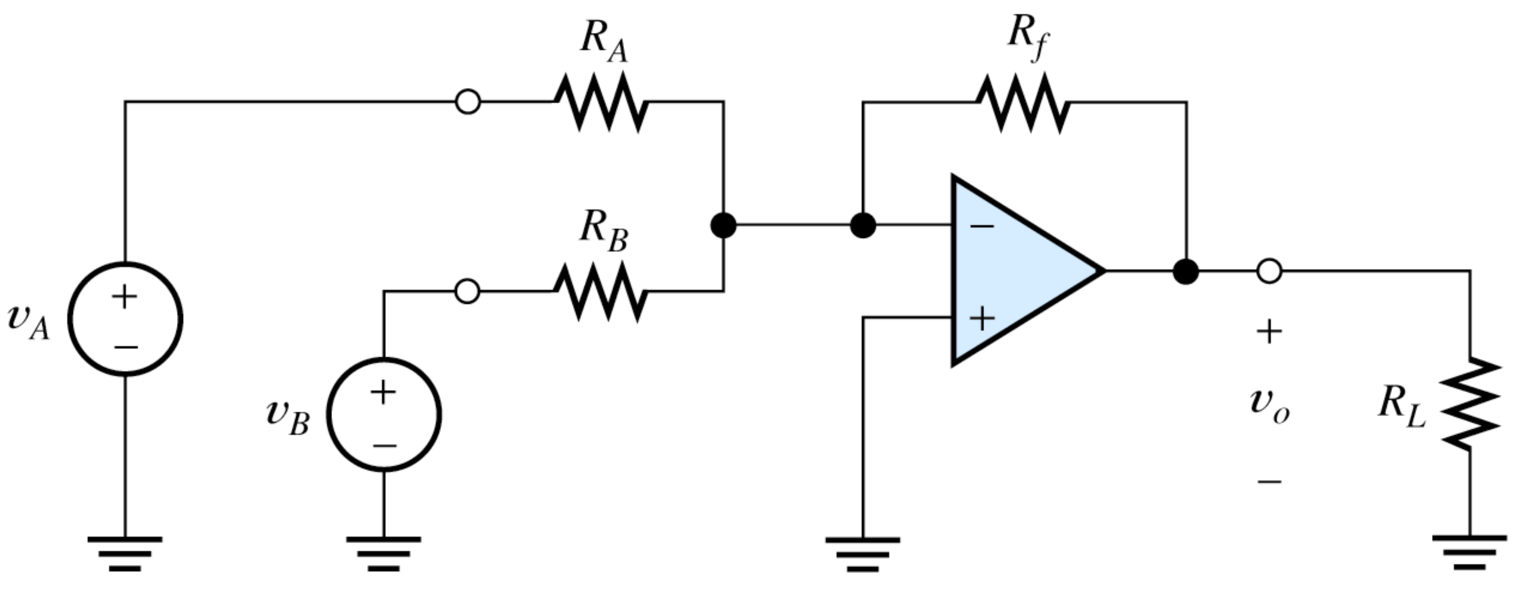
- **Voltage Follower**



$$A_v = \frac{v_o}{v_{\mathbf{in}}} = 1 + \frac{R_2}{R_1} = 1 + \frac{0}{\infty} = 1$$

Operational Amplifier (Op-amp)

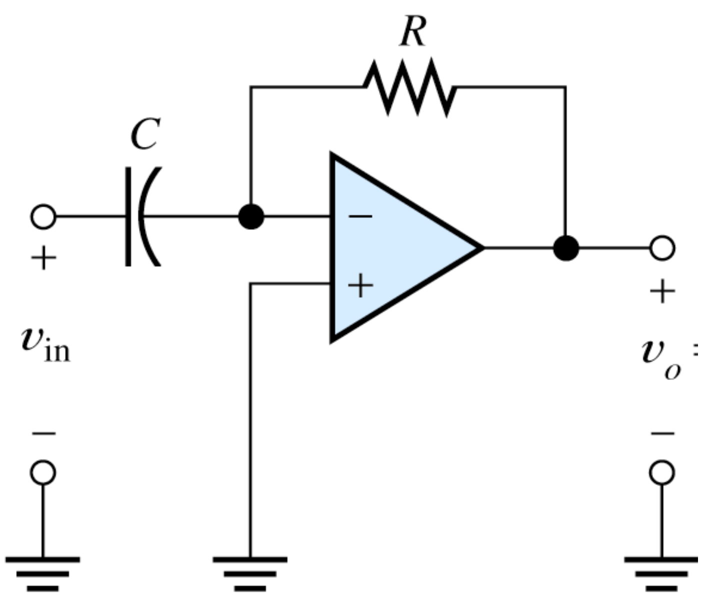
- **Summing Amplifier**



$$V_{out} = -R_f \times \left(\frac{V_A}{R_A} + \frac{V_B}{R_B} + \cdots + \frac{V_n}{R_n} \right)$$

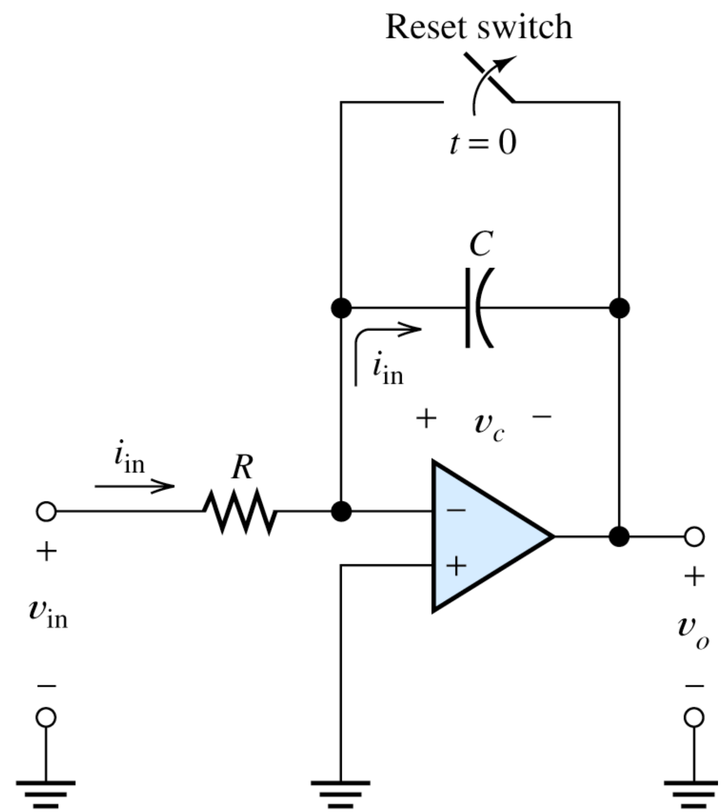
Operational Amplifier (Op-amp)

- **Differentiators**



$$v_o(t) = -RC\frac{dv_{in}}{dt}$$

- **Integrators**



$$v_o(t) = -\frac{1}{RC}\int_0^t v_{in}(t)dt$$