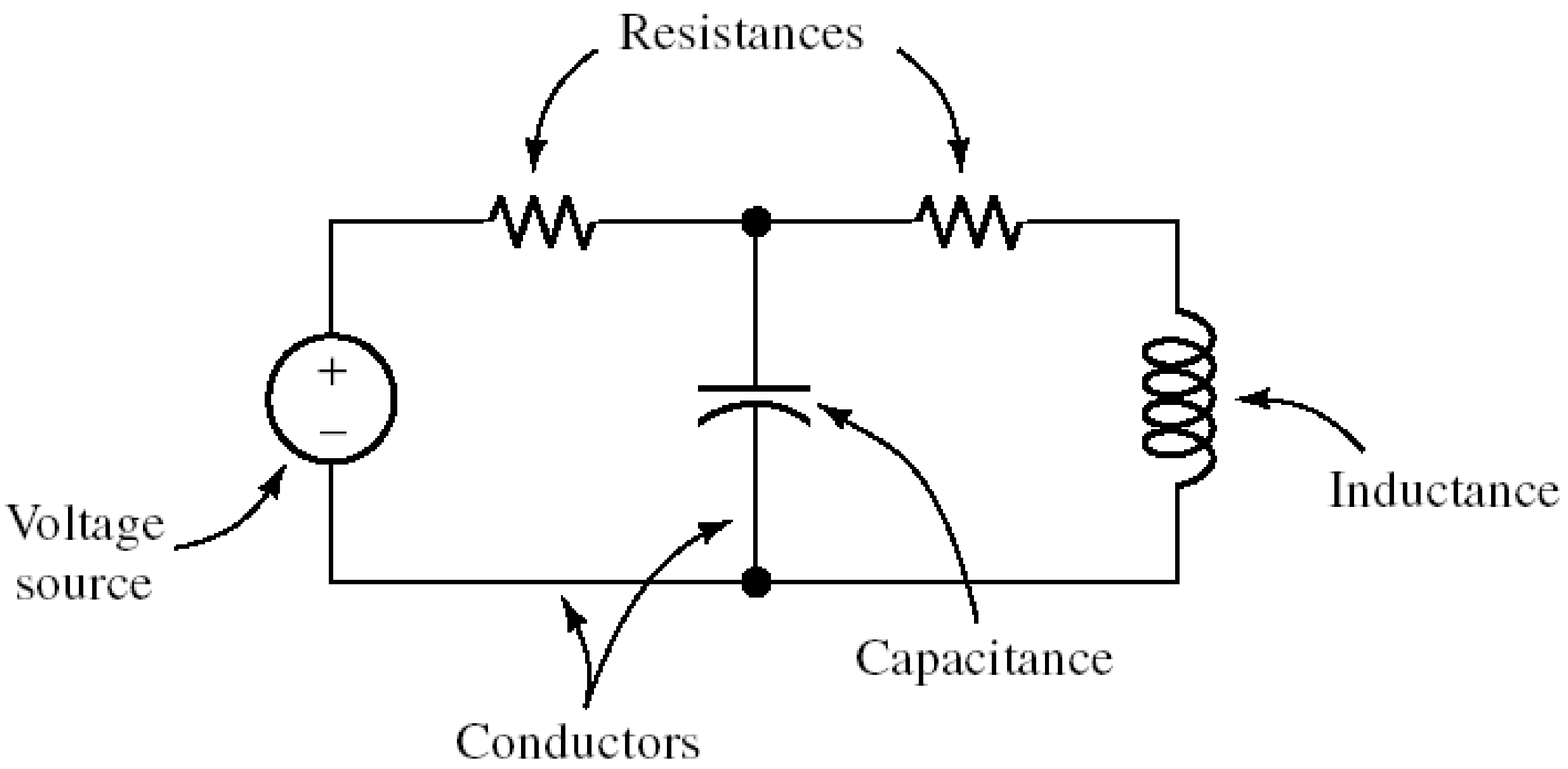
Appendix

DC Circuits
Capacitors and Inductors
AC Circuits
Operational Amplifiers

Circuit Elements

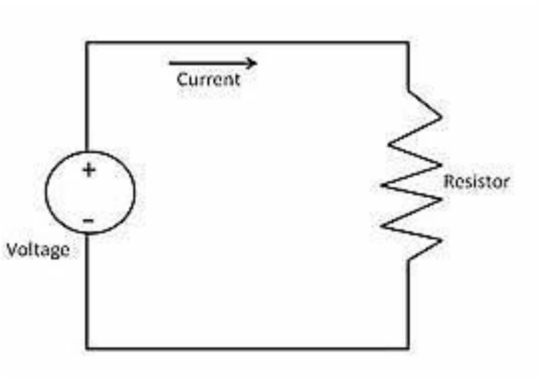


An electrical circuit consists of circuit elements such as voltage sources, resistances, inductances and capacitances that are connected in closed paths by conductors

DC Circuits

- **Current:**
  - The rate of motion of charge in a circuit
  - Symbol: I (or sometimes i).
  - SI units: C/s = ampere (A)
  - “Conventional current”
    - Assumed to consist of the motion of positive charges.
    - Conventional current flows from higher to lower potential.
  - AC/DC
    - Direct current (DC) flows in one direction around the circuit
    - Alternating current (AC) “sloshes” back and forth (time-varying that changes its sign periodically)

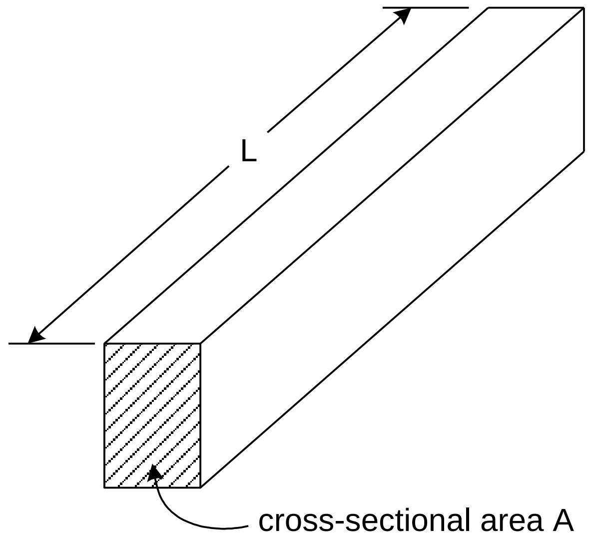
$$I = \frac{q}{t} = \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$$



DC Circuits

- **Ohm's Law and Resistance:**
  - The current that flows through an object is directly proportional to the voltage applied across the object
  - The constant of proportionality, R, is called the resistance of the object.
  - SI unit of resistance: ohm (Ω)
  - Resistance depends on the geometry of the object, and a property, resistivity, of the material from which it is made
  - Resistivity symbol: ρ
  - SI units of resistivity: ohm m (Ω m)

$$V = IR$$



$$R = \rho \frac{L}{A}$$

DC Circuits

- **Electrical Power**
  - Power is the time rate of doing work
  - Voltage is the work done per unit charge
  - Current is the time rate at which charge goes by
  - Combining

  - Ohm's Law substitutions allow us to write several equivalent expressions for power

  - Regardless of how specified, power always has SI units of watts (W)

$$P = \frac{W}{t}$$

$$V = \frac{W}{q}$$

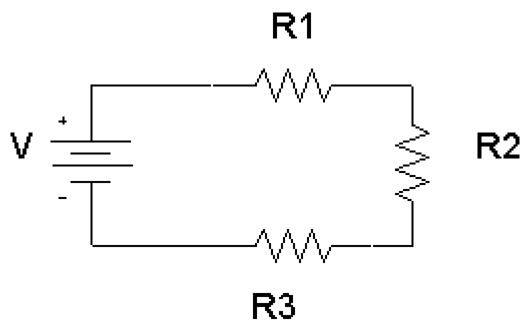
$$P = \frac{W}{t} = \frac{W}{q} \cdot \frac{q}{t} = VI$$

$$I = \frac{q}{t}$$

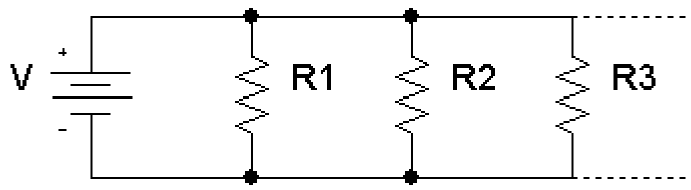
$$P = VI = I^2 R = \frac{V^2}{R}$$

DC Circuits

- **Series Connection**
  - A circuit, or a set of circuit elements, are said to be connected "in series" if there is only one electrical path through them.
  - The same current flows through all series-connected elements. (Equation of continuity)
- **Parallel Connection**
  - A circuit, or a set of circuit elements, are said to be connected "in parallel" if the circuit current is divided among them.
  - The same potential difference exists across all parallel-connected elements.



$$R_{eq} = R_1 + R_2 + \ldots + R_n$$

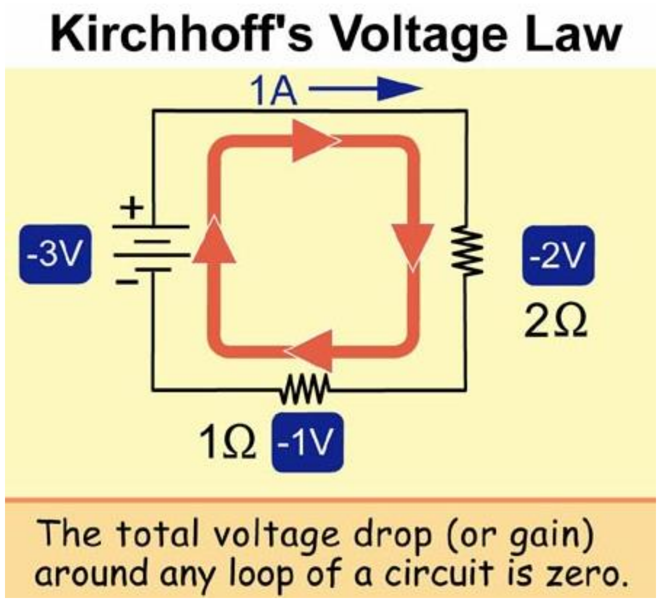


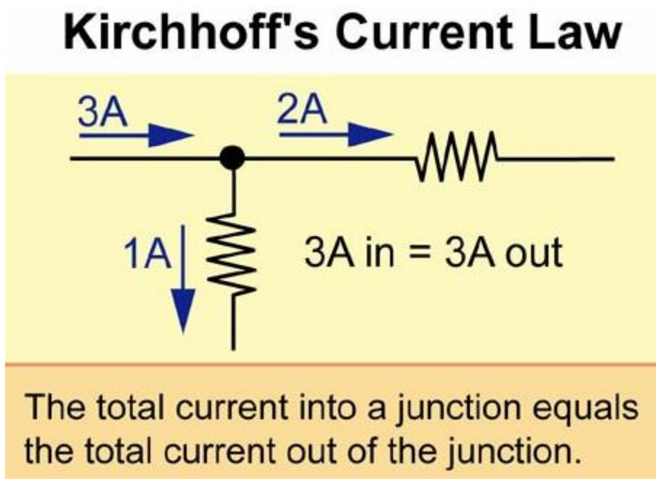
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

DC Circuits

Kirchoff’s Laws

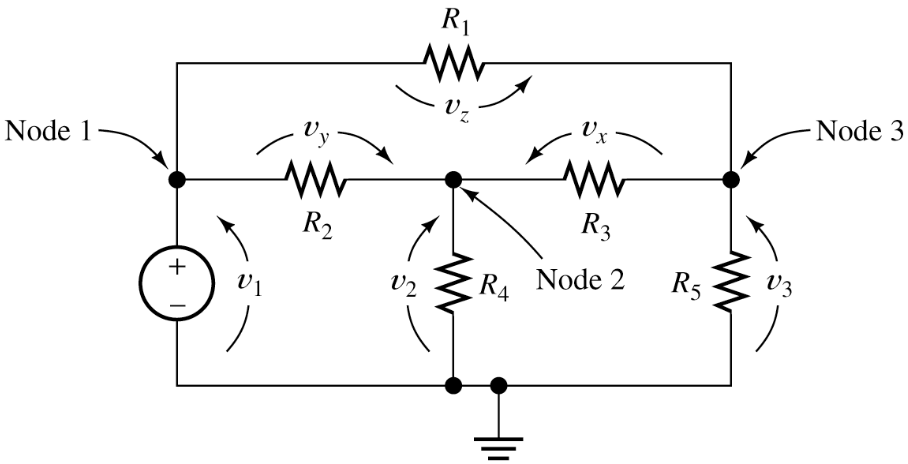
- **The Voltage Law:**
  - Around any closed loop in a circuit, the sum of the potential changes must equal zero.
  - (Energy conservation)
- **The Current Law:**
  - At any point in a circuit, the total of the currents flowing into that point must be equal to the total of the currents flowing out of that point.
  - (Charge conservation; equation of continuity)





DC Circuits

- **Node Voltage Analysis**



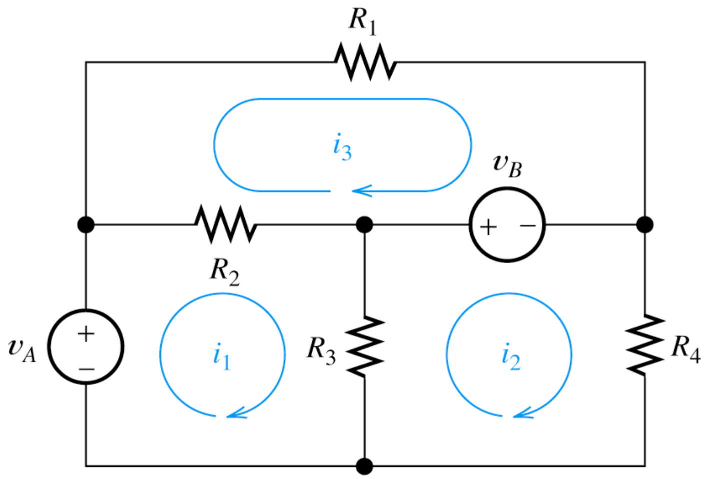
$$v_1 = v_s$$

From Kirchoff's Current Law

$$\frac{v_2 - v_1}{R_2} + \frac{v_2}{R_4} + \frac{v_2 - v_3}{R_3} = 0$$

$$\frac{v_3 - v_1}{R_1} + \frac{v_3}{R_5} + \frac{v_3 - v_2}{R_3} = 0$$

- **Mesh Current Analysis**



From Kirchoff's Voltage Law

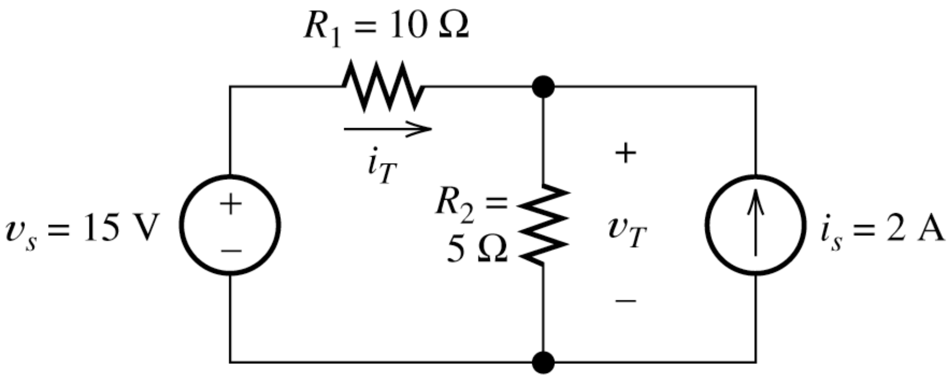
$$(i_1 - i_3)R_2 + (i_1 - i_2)R_3 - v_A = 0$$

$$(i_1 - i_2)R_3 - v_B - i_2 R_4 = 0$$

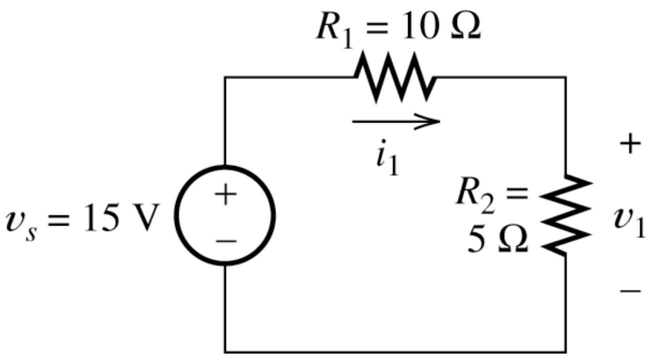
$$(i_1 - i_3)R_2 - i_3 R_1 + v_B = 0$$

DC Circuits

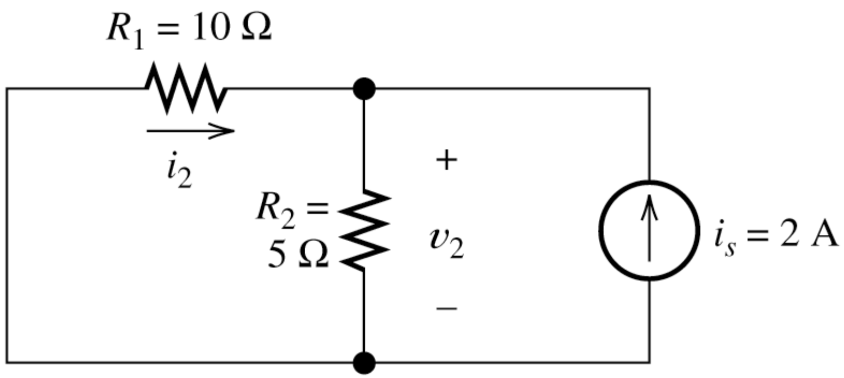
- **Superposition Principle**
  - The superposition principle states that the total response is the sum of the responses to each of the independent sources acting individually



(a) Original circuit



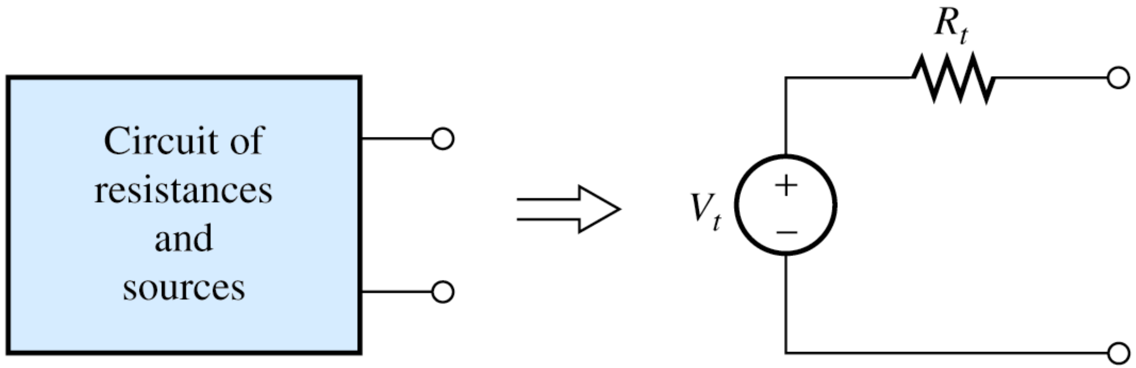
(b) Circuit with only the voltage source active

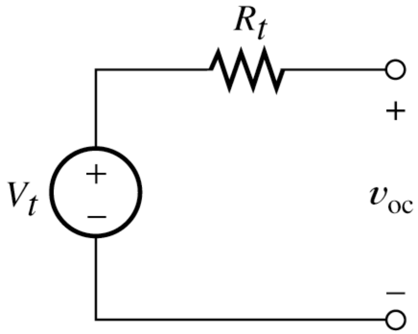


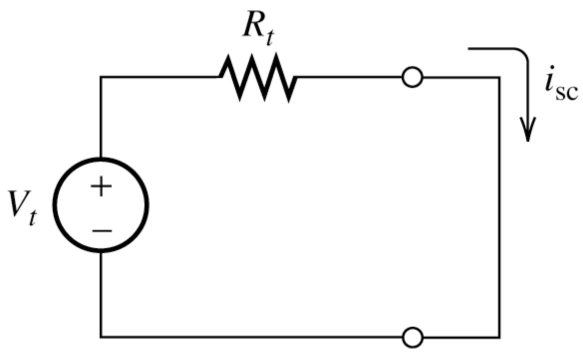
(c) Circuit with only the current source active

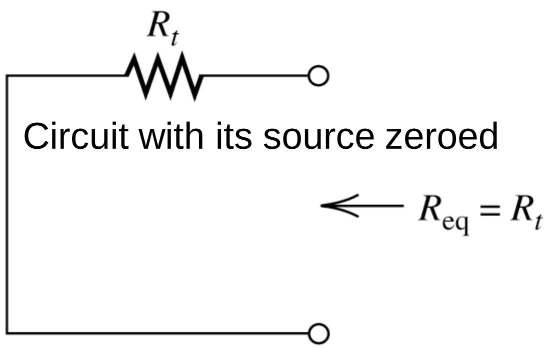
DC Circuits

- **Thévenin Equivalent Circuits**









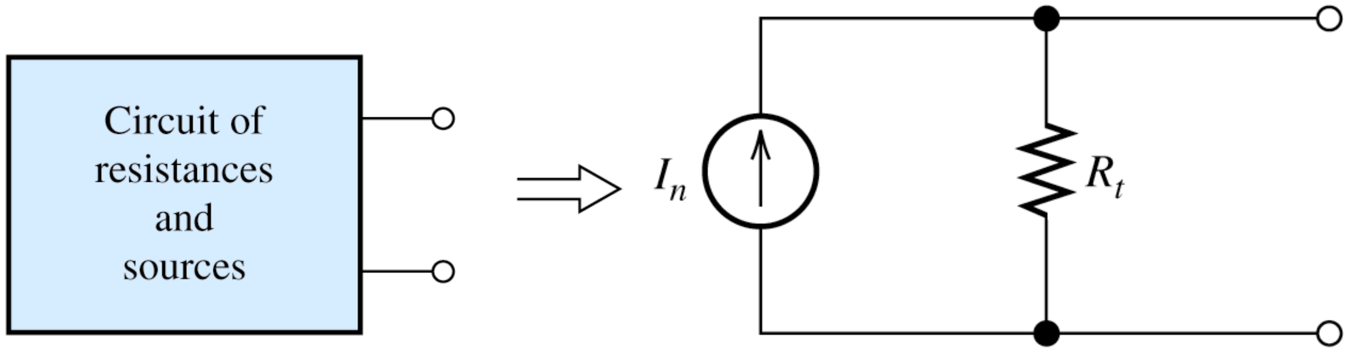
$$V_t = v_{oc}$$

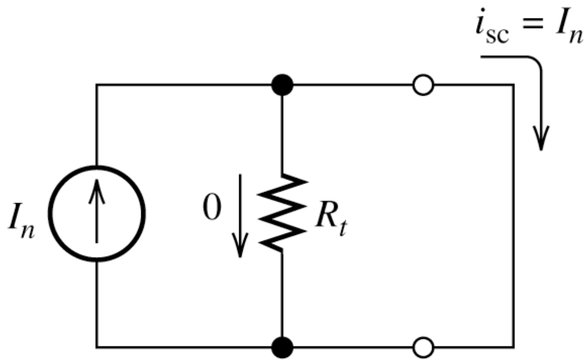
$$R_t = \frac{v_{oc}}{i_{sc}}$$

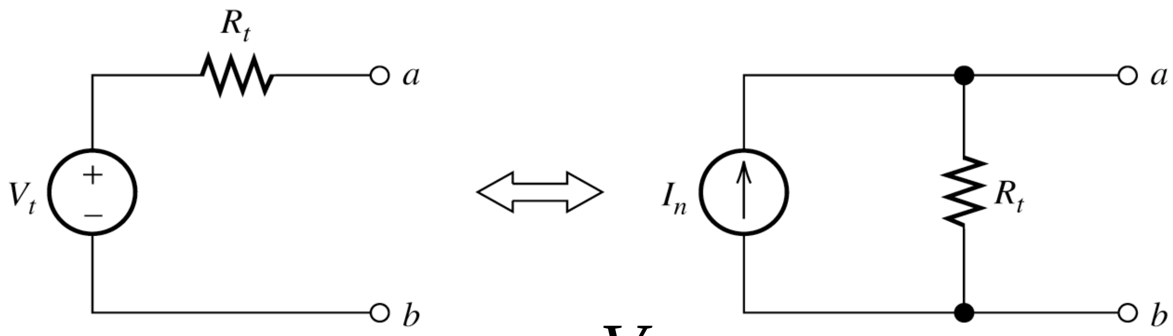
$$R_t = R_{eq}$$

DC Circuits

- **Norton Equivalent Circuits**





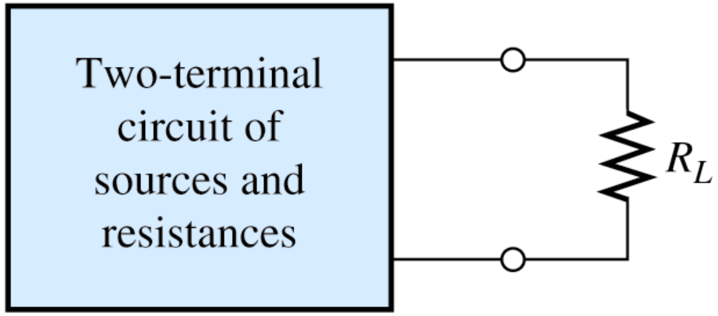


$$I_n = i_{sc}$$

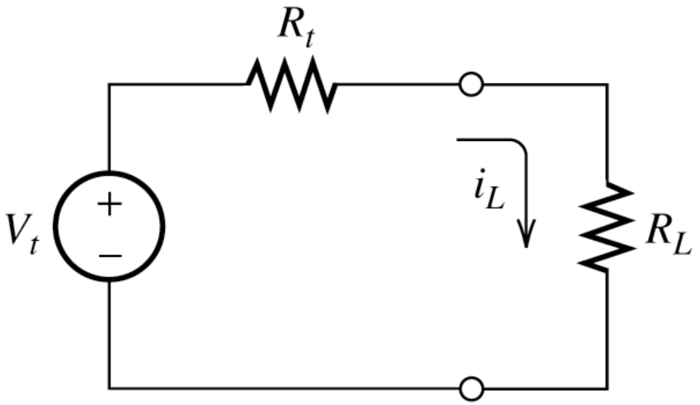
$$I_n = \frac{V_t}{R_t}$$

DC Circuits

- **Maximum Power Transfer**



(a) Original circuit with load

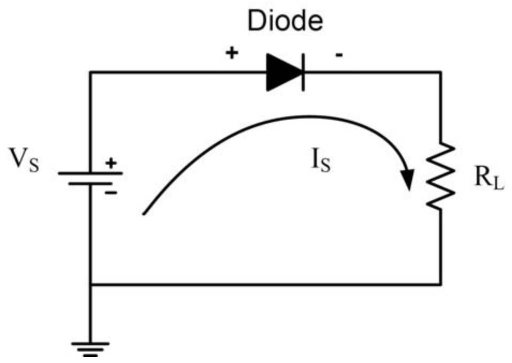


(b) Thévenin equivalent circuit with load

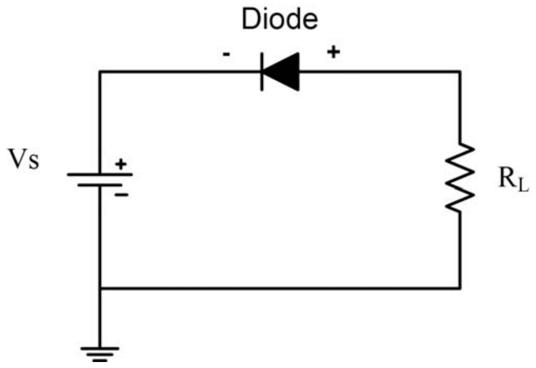
$$I_L = \frac{V_t}{R_t + R_L} \qquad P_L = i_L^2 R_L = \left(\frac{V_t}{R_t + R_L}\right)^2 R_L \qquad \frac{dP_L}{dR_L} = 0 \rightarrow R_L = R_t$$

DC Circuits

- **Diode**
  - Semiconductor device
  - Positive and negative polarities

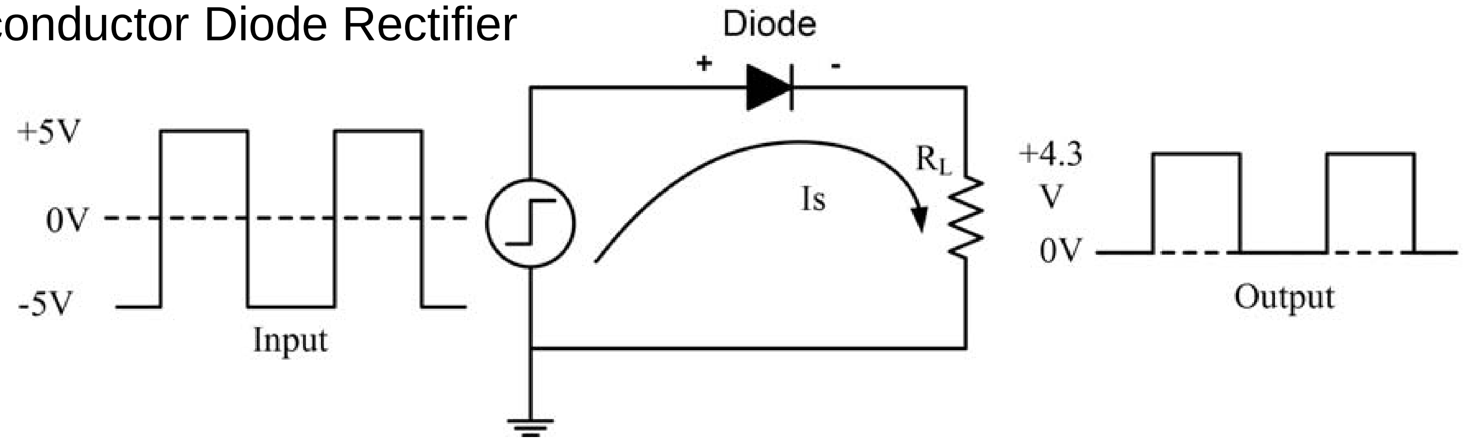


A forward-biased diode circuit



A reverse-biased diode circuit

Semiconductor Diode Rectifier



A bipolar square wave applied to a forward-biased diode circuit

Summary Table of DC Concepts

Device/Measurement	Action	Result
Series resistors	Simply add them together	R_T is larger than any single resistor value
Parallel resistors	Add them after inverting each value of the resistor, then invert the result	R_T is less than any single resistor value
Voltage drop in a series resistive circuit	The sum of drops equals the total source voltage	As the resistor value goes up, the value of the voltage drop also increases making them directly proportional
Currents in parallel circuits	Sum of branch currents equals the total current	As the resistor value goes up, the current value goes down making them inversely proportional
Capacitor	Voltage charges in a capacitor	Time constant $= R \times C$
Inductor	Current builds up in an inductor	Time constant $= \frac{L}{R}$

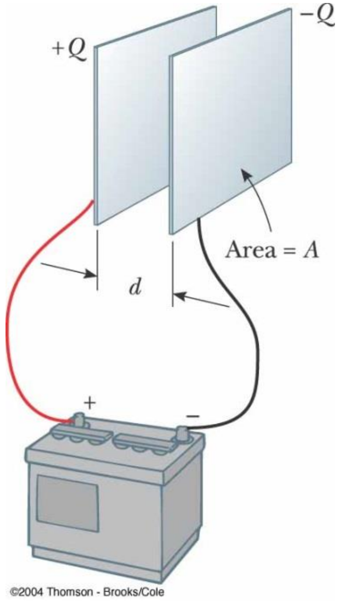
Capacitors and Inductors

- **Capacitance**
  - Ability to store charge
  - The SI unit of capacitance is the farad:
    - 1 farad = 1 F = 1 Coulomb/Volt
  - For a given charge, a capacitor with a larger capacitance will have a greater potential difference

$$Q = \frac{\varepsilon_0 A}{d}\Delta V = C\Delta V$$

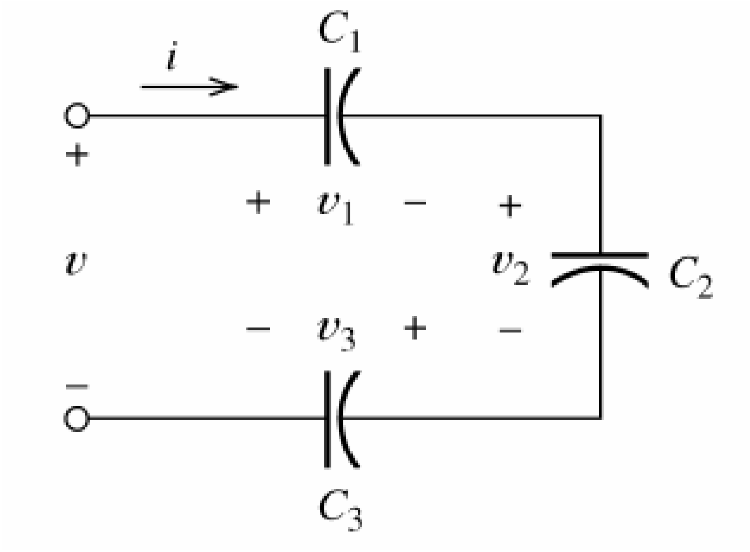
$$C = \frac{\varepsilon_0 A}{d}$$

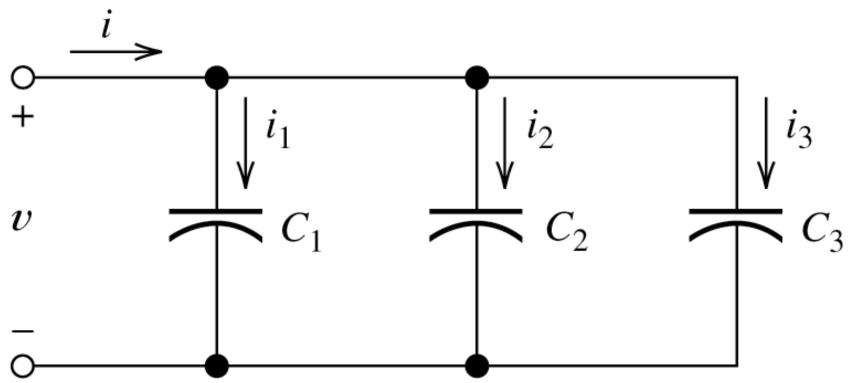
$$C = \frac{k\varepsilon_0 A}{d}$$ (with dielectric)



Capacitors and Inductors

- **Like resistors, capacitors in circuits can be connected in series, in parallel, or in more-complex networks containing both series and parallel connections.**



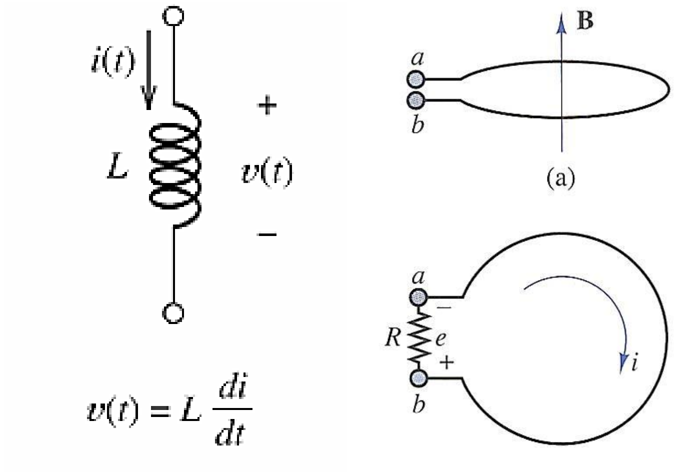


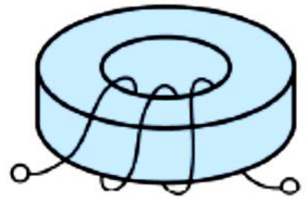
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + .. + \frac{1}{C_n}$$

$$C_{eq} = C_1 + C_2 + ... + C_n$$

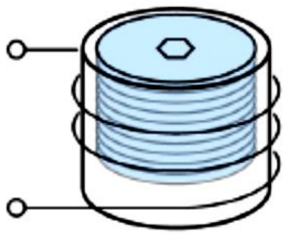
Capacitors and Inductors

- **Inductance**
  - Ability to store magnetic energy
  - The polarity of the voltage is such as to oppose the change in current (Lenz’s law).
  - Inductance, unit: henry [H]

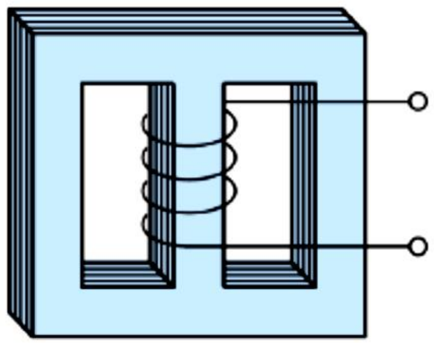




(a) Toriodal inductor



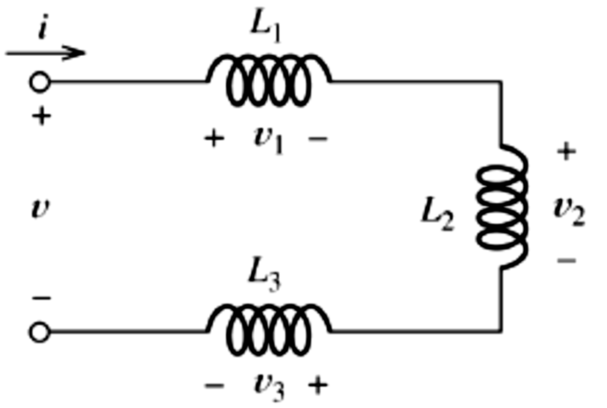
(b) Coil with an iron-oxide slug that can be screwed in or out to adjust the inductance



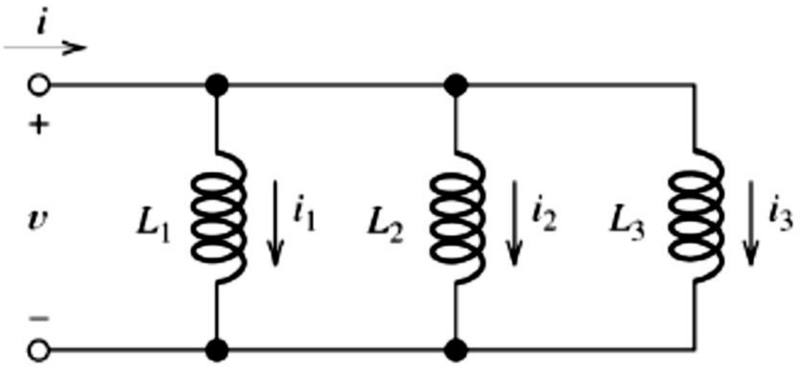
(c) Inductor with a laminated iron core

Capacitors and Inductors

- **Like resistors, inductors in circuits can be connected in series, in parallel, or in more-complex networks containing both series and parallel connections.**



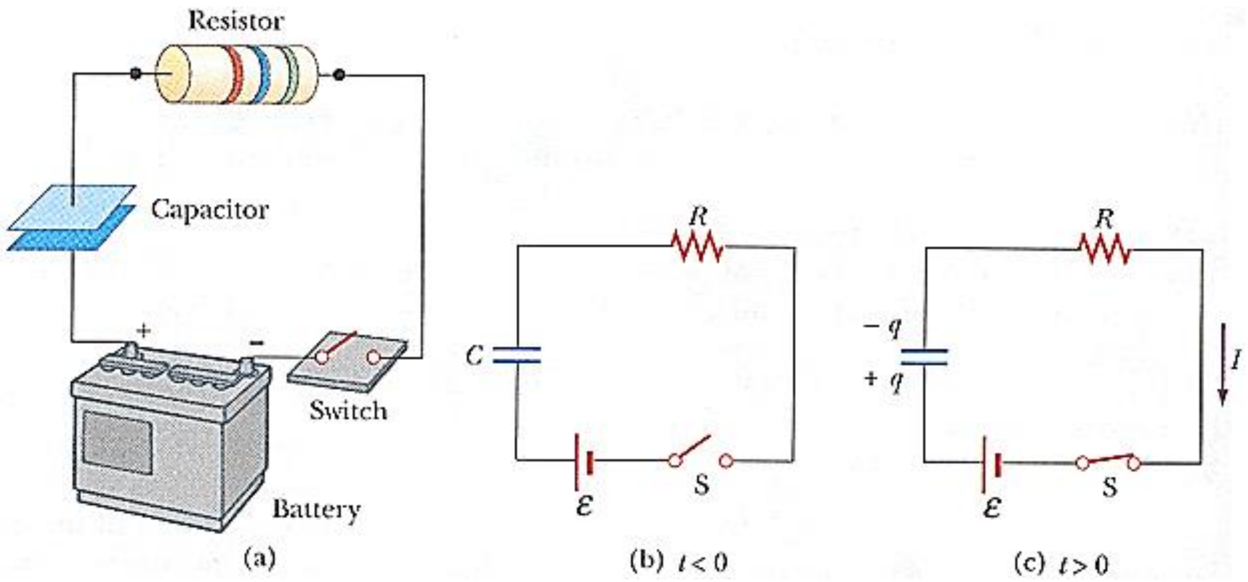
$$L_{eq} = L_1 + L_2 + L_3$$



$$\frac{1}{L_{eq}} = \frac{1}{L_1 + L_2 + L_3}$$

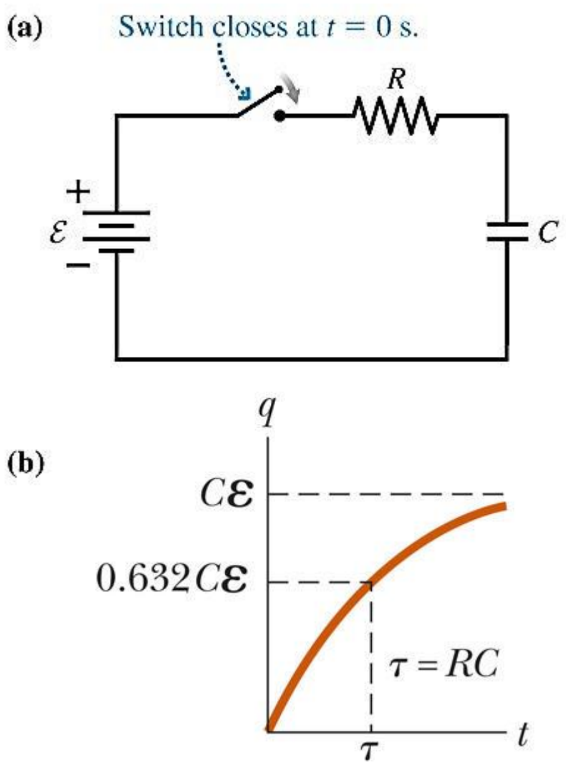
RC Circuits

- **A capacitor connected in series with a resistor is part of an RC circuit.**
  - Resistance limits charging current
  - Capacitance determines ultimate charge
- **Unlike a battery, a capacitor cannot provide a constant source of potential difference.**
  - This value is constantly changing as the charge leaves the plate.
  - Current due to a discharging capacitor is finite and changes over time.



RC Circuits

- **Charging the capacitor**
  - Assume no initial charge in the capacitor
  - At the instant the source is connected, the capacitor starts to charge.
    - The capacitor continues to charge until it reaches its maximum charge (Q = Cε)
  - Once the capacitor is fully charged, the current in the circuit is zero.
    - The potential difference across the capacitor matches that supplied by the battery
  - The charge on the capacitor increases exponentially with time
    - τ is the time constant
    - τ = RC



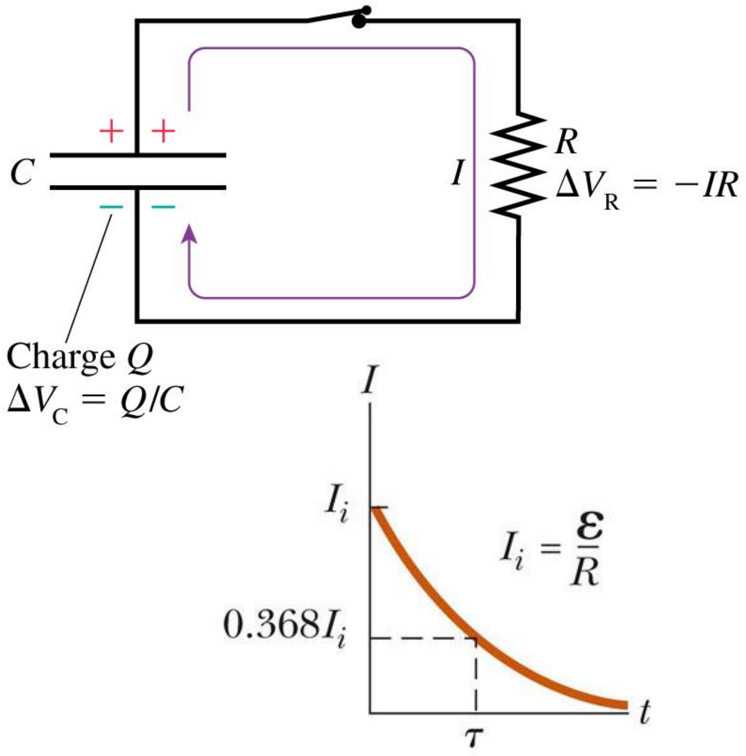
$$q(t) = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right) = Q_0\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$I(t) = \frac{dq}{dt} = \frac{Ce}{RC}e^{-\frac{t}{RC}} = I_o e^{-\frac{t}{\tau}}$$

RC Circuits

- **Discharging the capacitor**
  - Assume a fully charged capacitor
  - At the instant the switch closes, the capacitor starts to charge.
    - The capacitor continues to discharge until it reaches 0
  - Once the capacitor is fully discharged, the current in the circuit is zero.

  - The charge on the capacitor decreases exponentially with time
    - τ is the time constant
    - τ = RC



$$q(t) = C\varepsilon e^{-\frac{t}{RC}} = Q_0 e^{-\frac{t}{\tau}}$$

$$I(t) = \frac{dQ}{dt} = \frac{C\varepsilon}{RC} e^{-\frac{t}{\tau}} = I_0 e^{-\frac{t}{\tau}}$$

AC Circuits

- **Instantaneous voltage**
- **Instantaneous current**
  - θ is the phase angle
- **In phasor form**
- **Impedance**
  - In series
  - In parallel

$v(t) = V_{max} \sin \omega t$

$i(t) = I_{max} \sin (\omega t - \theta)$ $I_{max} = V_{max} / |Z|$

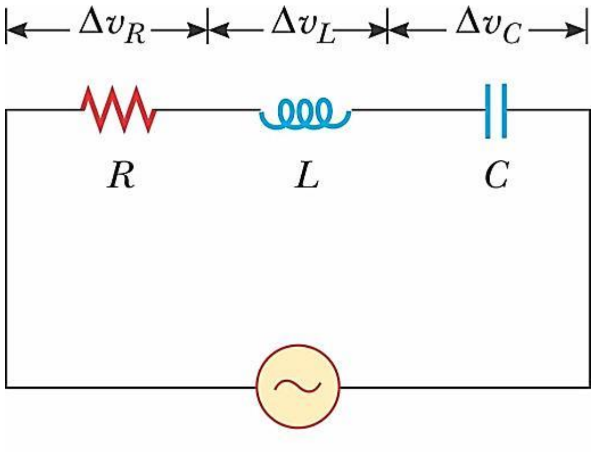
$\theta = \tan^{-1}(X/R)$ $Z = V / I$ $|Z| \equiv \sqrt{R^2 + (X_L - X_C)^2}$

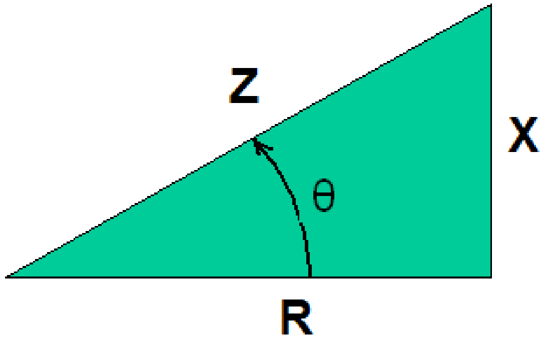
$V = V_{rms} \angle 0$ $I = I_{rms} \angle \theta$ $V_{rms} = V_{max}/\sqrt{2}$, $I_{rms} = I_{max}/\sqrt{2}$

$Z = R + jX$ $X = X_L - X_c$

$Z_{eq} = (R_1+R_2)+j(X_1+X_2)$

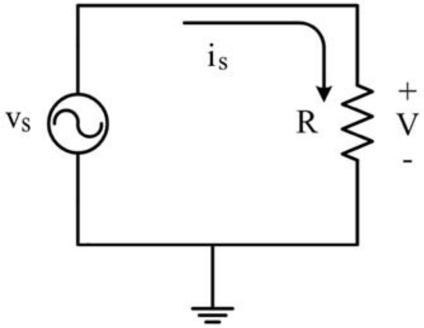
$1/Z_{eq} = 1/(R_1 + jX_1) + 1/(R_2 + jX_2)$





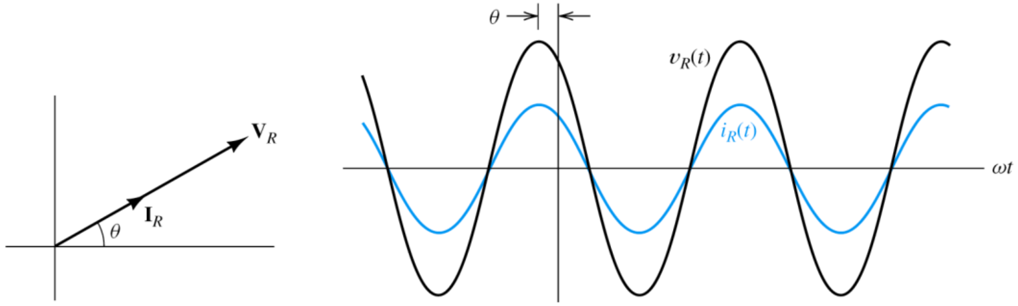
AC Circuits

Voltage and current waveforms in a resistive circuit



$$\mathbf{V}_R = R\mathbf{I}_R$$

$$Z_R = R$$

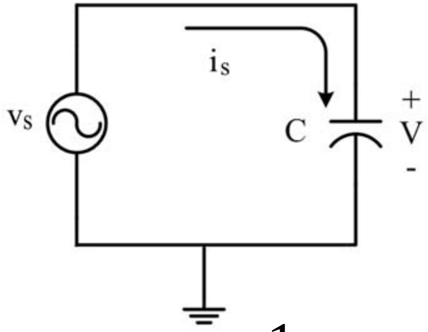


(a) Phasor diagram

(b) Current and voltage versus time

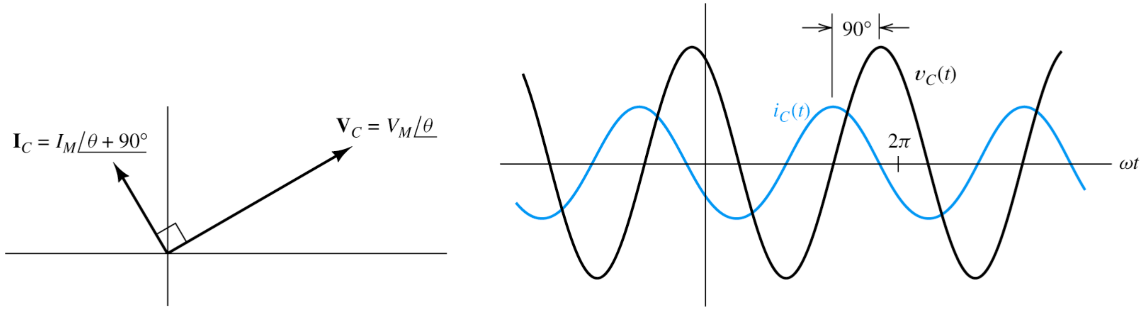
Figure 5.9 For a pure resistance, current and voltage are in phase.

Voltage and current waveforms in a capacitive circuit



$$\mathbf{V}_C = Z_C\mathbf{I}_C$$

$$Z_C = -j\frac{1}{\omega C} = \frac{1}{j\omega C} = \frac{1}{\omega C}\angle -90^\circ$$



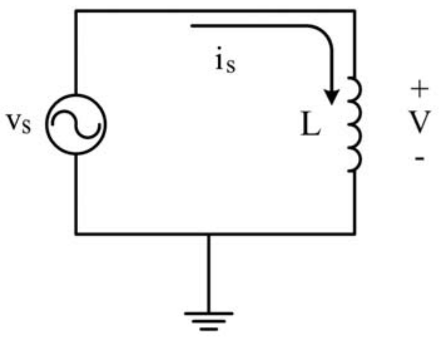
(a) Phasor diagram

(b) Current and voltage versus time

Figure 5.8 Current leads voltage by 90° in a pure capacitance.

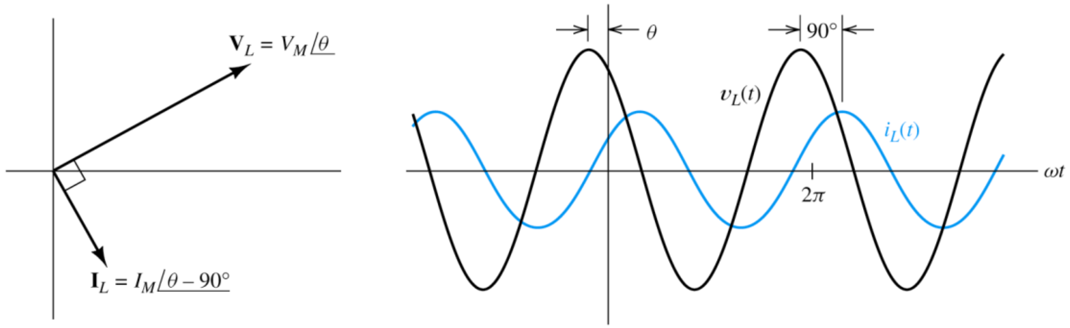
AC Circuits

Voltage and current waveforms in an inductive circuit



$$\mathbf{V}_L = Z_L \mathbf{I}_L$$

$$\mathbf{V}_L = j\omega L \times \mathbf{I}_L$$

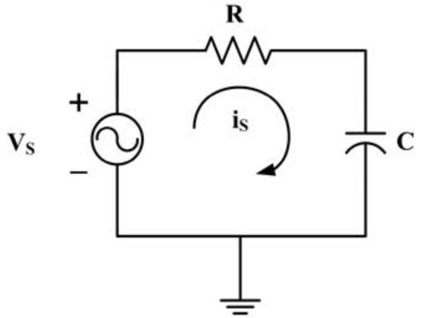


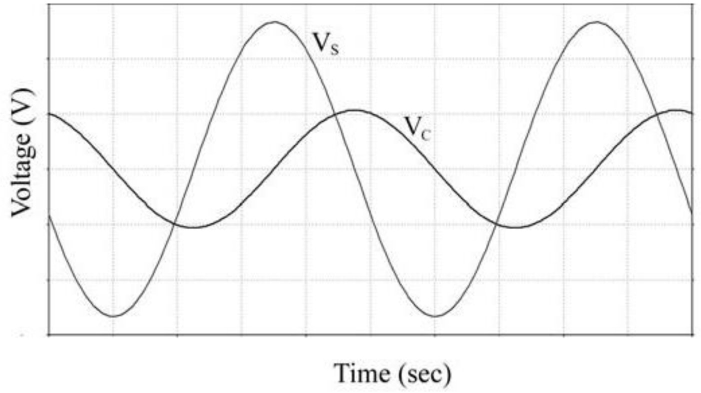
(a) Phasor diagram (b) Current and voltage versus time

Figure 5.7 Current lags voltage by 90° in a pure inductance.

$$Z_L = j\omega L = \omega L \angle 90^\circ$$

Voltage and current waveforms in an RC series circuit





AC Circuits

- Power can be expressed in rectangle form
- P- real power
- Q–reactive power
- Power factor
- Maximum Power Transfer

$$S = P + jQ$$

$$P = V_{rms} I_{rms} \cos(\theta) = I^2_{rms} R$$

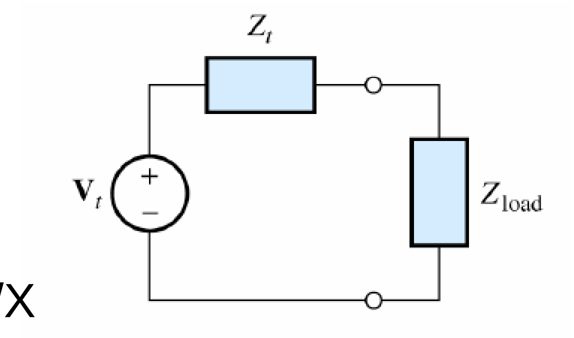
$$Q = V_{rms} I_{rms} \sin(\theta) = V^2_{rms}/X$$

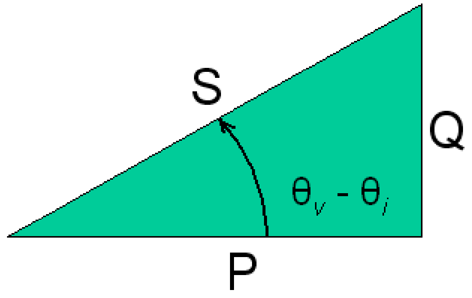
$$S^2 = P^2 + Q^2$$

$$PF = \cos(\theta) = \cos(\theta_v - \theta_i)$$

$$\cos(\theta) = 1 \rightarrow \theta = 0$$

$$Z_{load} = Z_t^*$$



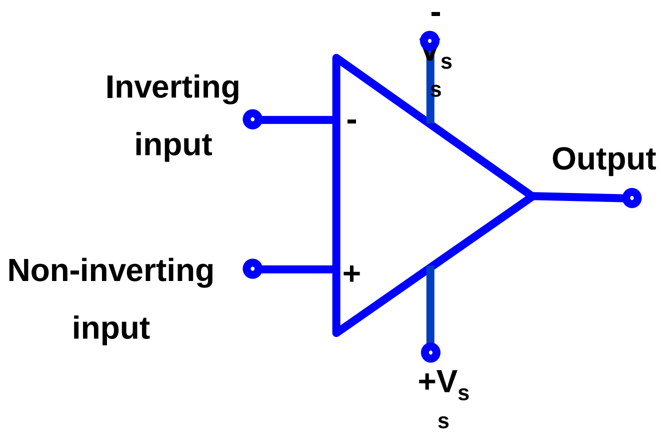


Summary Table of AC Concepts

Device/Measurement	Action	Result
Inductor	Inductive voltage leads the current by 90°	The inductor is open at high frequency
Capacitor	Capacitive current leads the voltage by 90°	The capacitor acts as a short at high frequency
Resistor	$R\ \Omega\angle 0°$	Real number
Capacitive reactance	$X_C\Omega\angle -90°$	$-J\ X_C\Omega$
Inductive reactance	$X_L\Omega\angle 90°$	$+J\ X_L\Omega$
Capacitive circuit	$R - J\ X_C$	$R\angle 0° + X_C\angle -90°$
Inductive circuit	$R + J\ X_L$	$R\angle 0° + X_L\angle 90°$

Operational Amplifier (Op-amp)

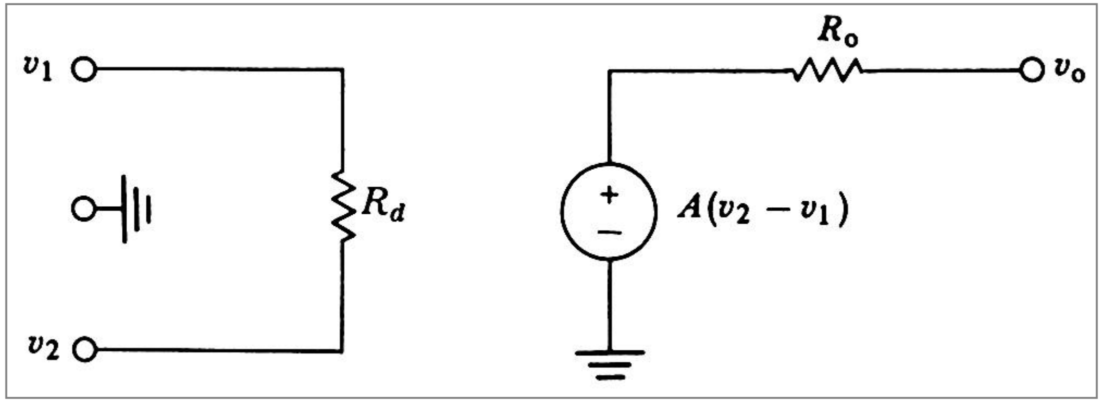
- **The op-amp is a device for increasing the power of a signal.**
  - It does this by taking power from a power supply and controlling the output to match the input signal shape but with a larger amplitude (Amplification).
- **The op-amp is used also to perform arithmetic operations (addition, subtraction, multiplication) with signals.**
- **The properties of the negative feedback loop determine the properties of the circuit containing an op-amp.**
- **It has two inputs: the inverting input (-) and the non-inverting input (+), and one output.**
- **It has usually two supplies (±Vss) but it can work with one.**



Symbol of op-amplifier

Operational Amplifier (Op-amp)

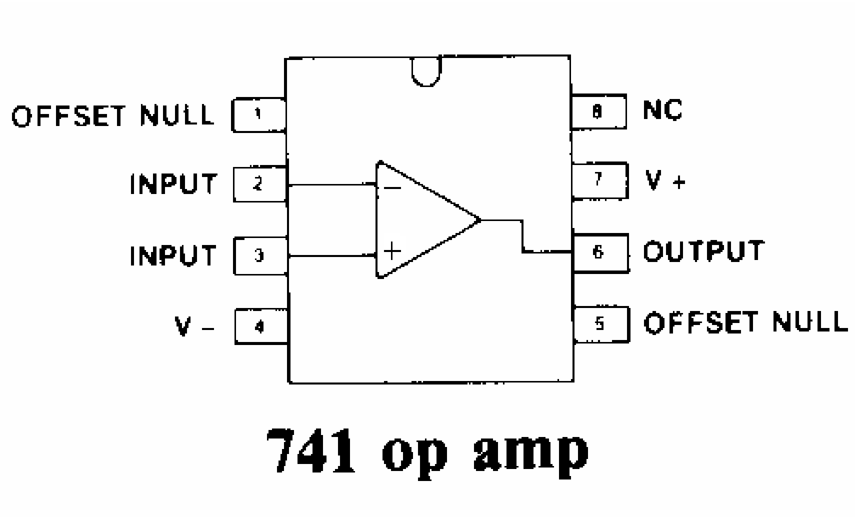
- **Ideal Op-Amp**
  - Op-amp equivalent circuit:

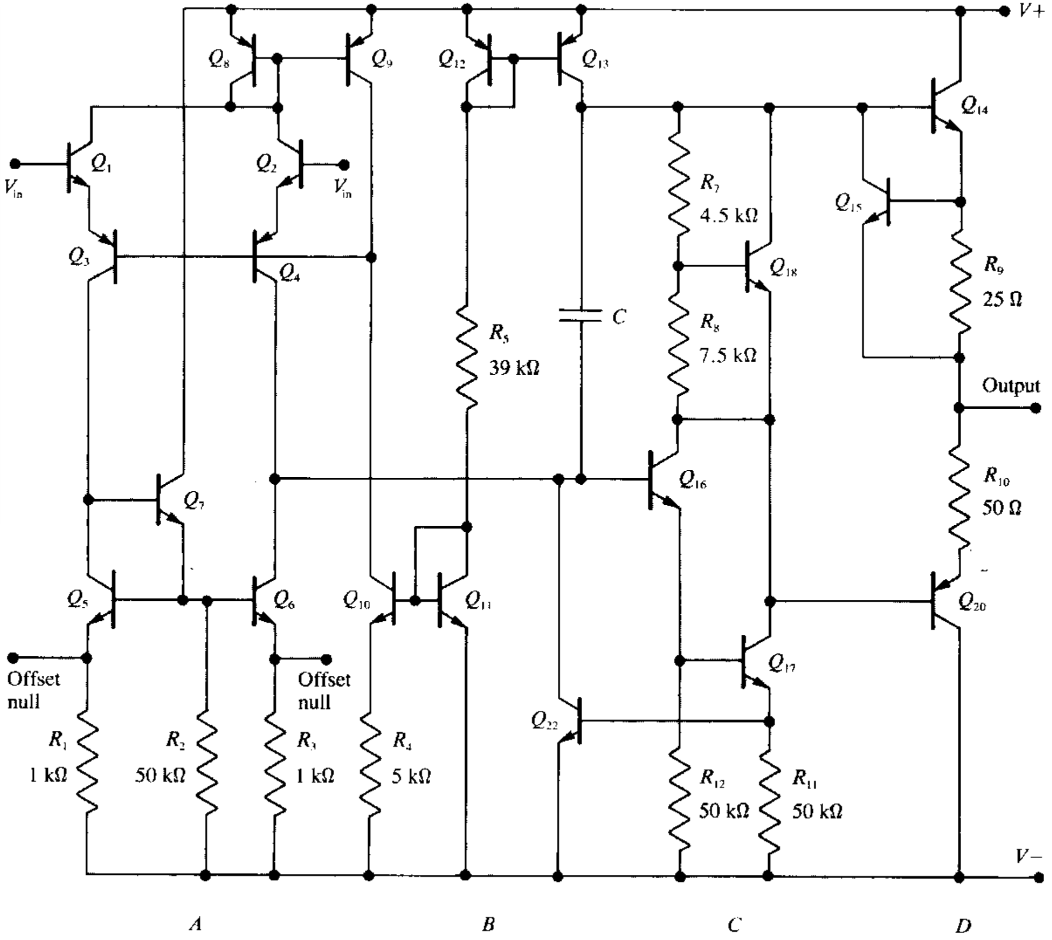


  - The two inputs are υ1 and υ 2.
  - A differential voltage between them causes current flow through the differential resistance Rd.
  - The differential voltage is multiplied by A (the open-loop gain of the op amp) to generate the output-voltage source
  - Any current flowing to the output terminal vo must pass through the output resistance Ro.

Operational Amplifier (Op-amp)

- **Inside the Op-Amp (IC-chip)**





20 transistors
11 resistors
1 capacitor

Operational Amplifier (Op-amp)

- **Real vs. Ideal Op-amp**

Parameter	Ideal Op Amp	Typical Op Amp
Open-loop voltage gain **A**	∞	$10^5 – 10^9$
Common mode voltage gain	0	10^{-5}
Frequency response **f**	∞	1- 20 MHz
Input impedance $\mathbf{Z_{in}}$	∞	10^6 Ω (bipolar) 10^9–10^{12} Ω (FET)
Output impedance $\mathbf{Z_{out}}$	0	100 – 1000 Ω

Operational Amplifier (Op-amp)

- **Summing Point Constraint**
  - In a negative feedback system, the ideal op-amp output voltage attains the value needed to force the differential input voltage and input current to zero.
- **Circuit solution**
  1. Verify that negative feedback is present.
  2. Assume that the differential input voltage and the input current of the op amp are forced to zero. (This is the summing-point constraint.)
  3. Apply standard circuit-analysis principles, such as Kirchhoff's laws and Ohm's law, to solve for the quantities of interest.

Operational Amplifier (Op-amp)

- **Applying the Summing Point Constraint**

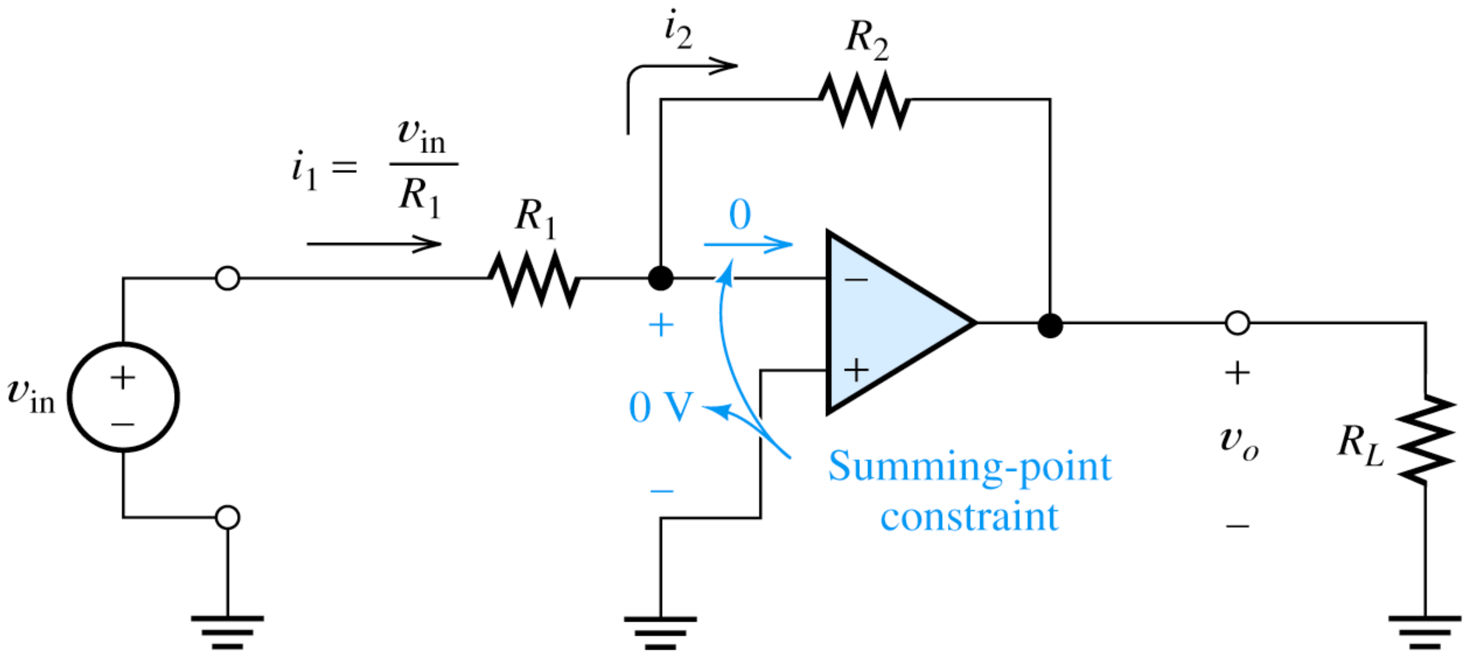
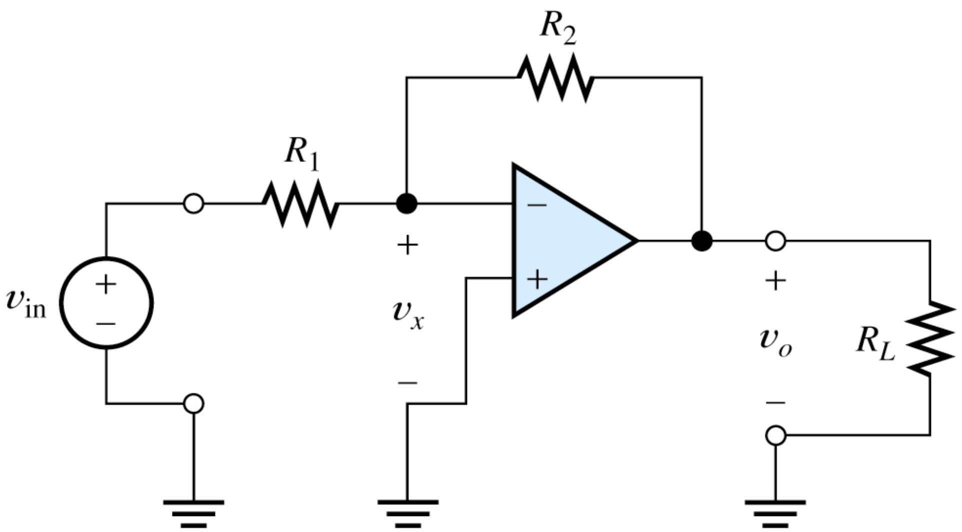


Figure 14.5 We make use of the summing-point constraint in the analysis of the inverting amplifier.

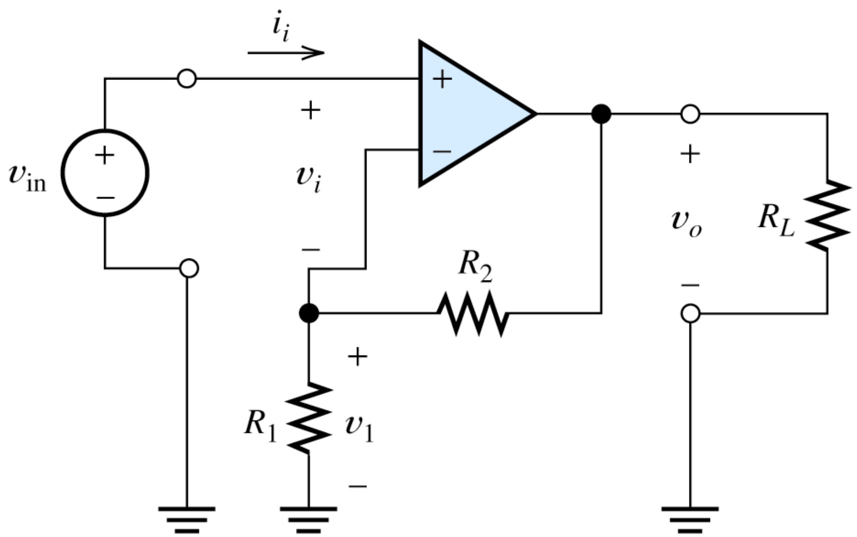
Operational Amplifier (Op-amp)

- **Inverting Amplifier**



$$A_v = \frac{v_o}{v_{in}} = -\frac{R_2}{R_1}$$

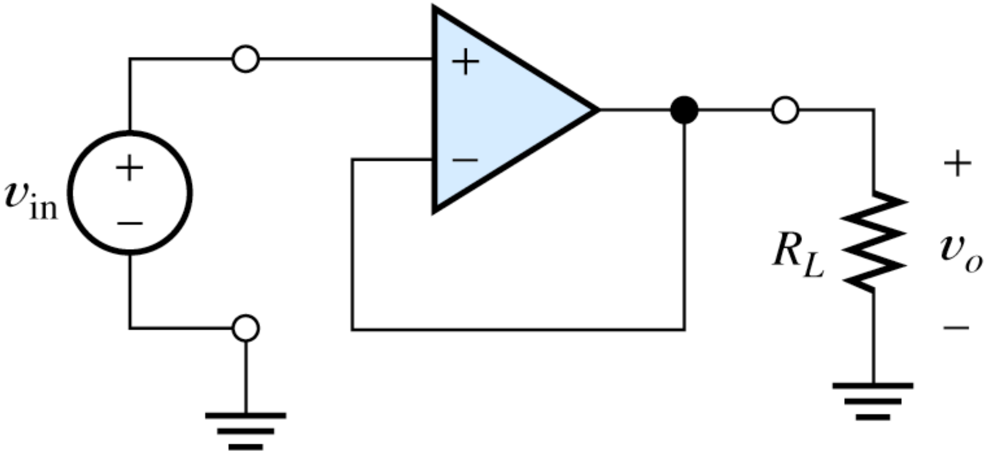
- **Non-inverting Amplifiers**



$$A_v = \frac{v_o}{v_{in}} = 1 + \frac{R_2}{R_1}$$

Operational Amplifier (Op-amp)

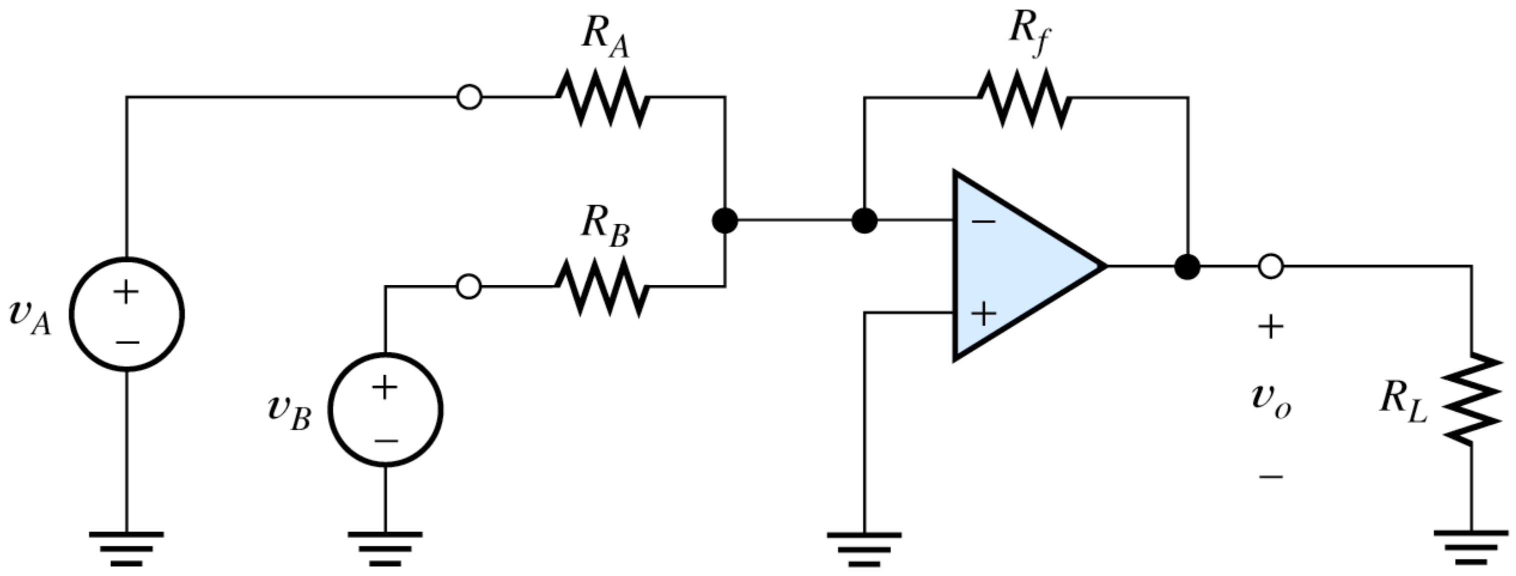
- **Voltage Follower**



$$A_v = \frac{v_o}{v_{in}} = 1 + \frac{R_2}{R_1} = 1 + \frac{0}{\infty} = 1$$

Operational Amplifier (Op-amp)

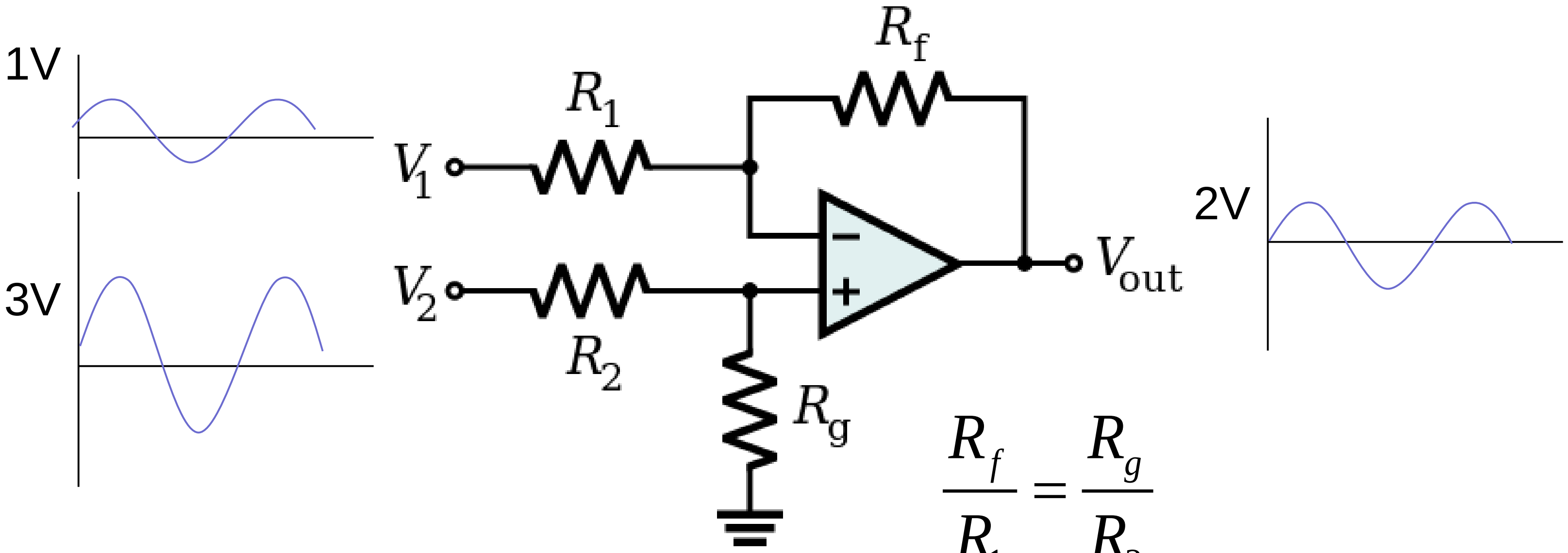
- **Summing Amplifier**



$$V_{out} = -R_f \times \left(\frac{V_A}{R_A} + \frac{V_B}{R_B} + \cdots + \frac{V_n}{R_n} \right)$$

Operational Amplifier (Op-amp)

- **Differential Amplifier**
  - In differential mode you can signals common to both input signals



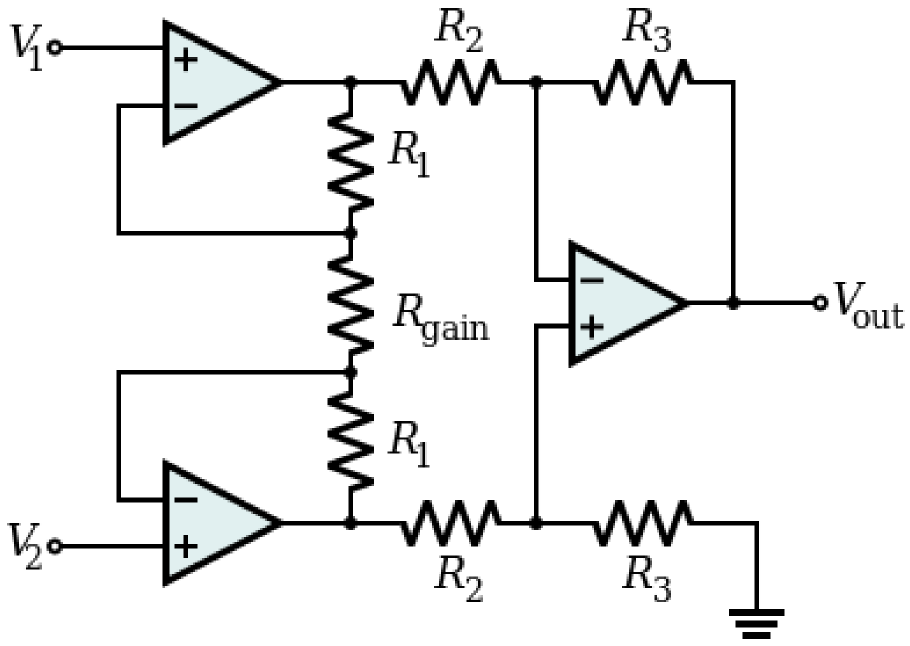
$$\frac{R_f}{R_1} = \frac{R_g}{R_2}$$

$$V_{out} = \frac{R_f}{R_1}\left(V_2 - V_1\right)$$

Operational Amplifier (Op-amp)

•Instrumentation Amplifier

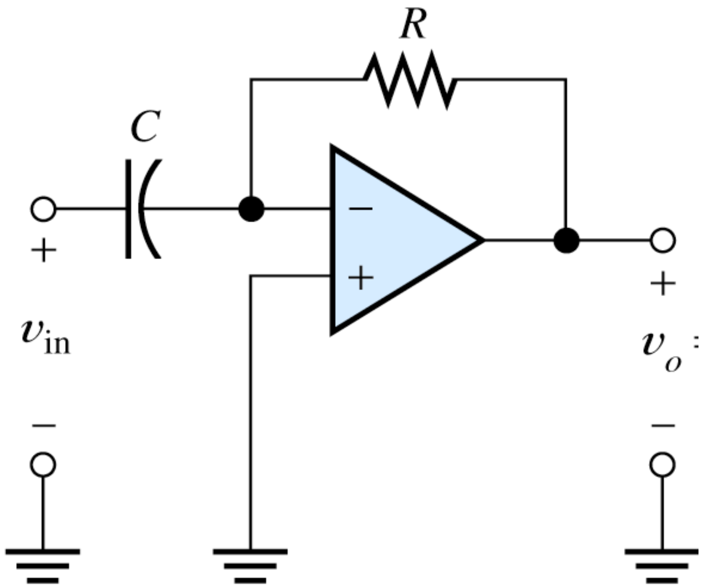
- High gain and high-input impedance.
- Composed of 2 amplifiers in noninverting format and a 3rd amplifier as a differential amplifier



$$\frac{V_{out}}{V_2 - V_1} = \left(1 + \frac{2R_1}{R_{gain}}\right)\left(\frac{R_3}{R_2}\right)$$

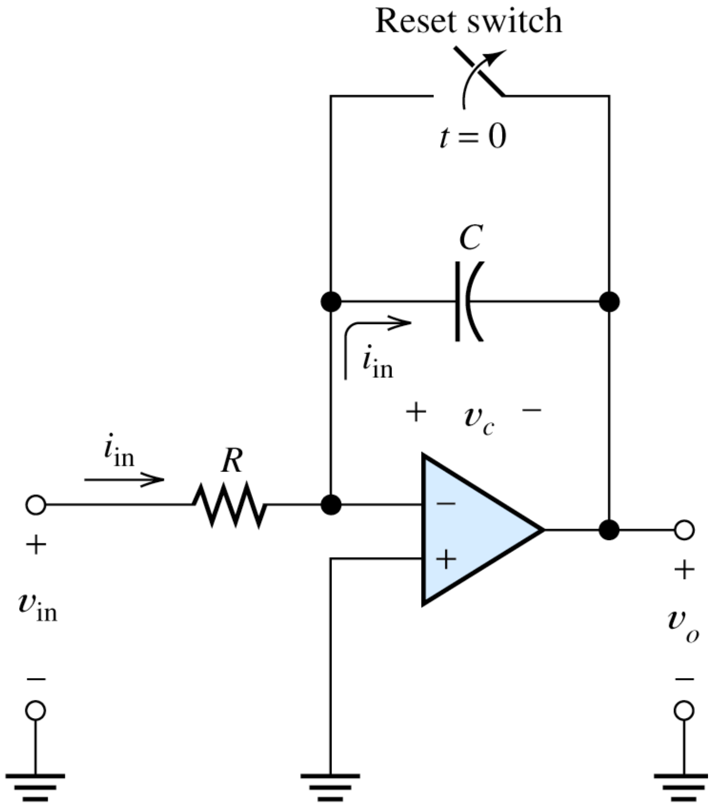
Operational Amplifier (Op-amp)

- Differentiators



$$v_o(t) = -RC\frac{dv_{in}}{dt}$$

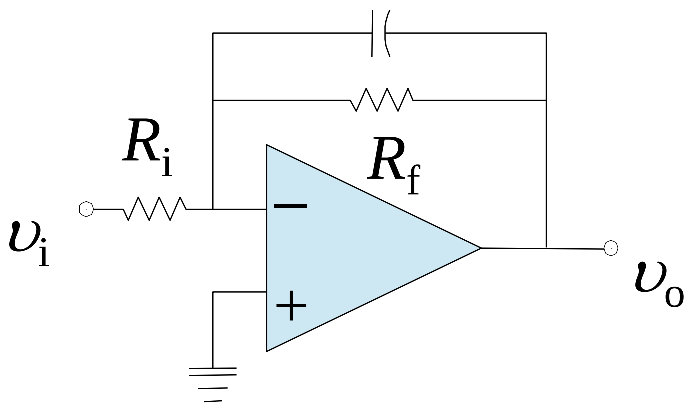
- Integrators

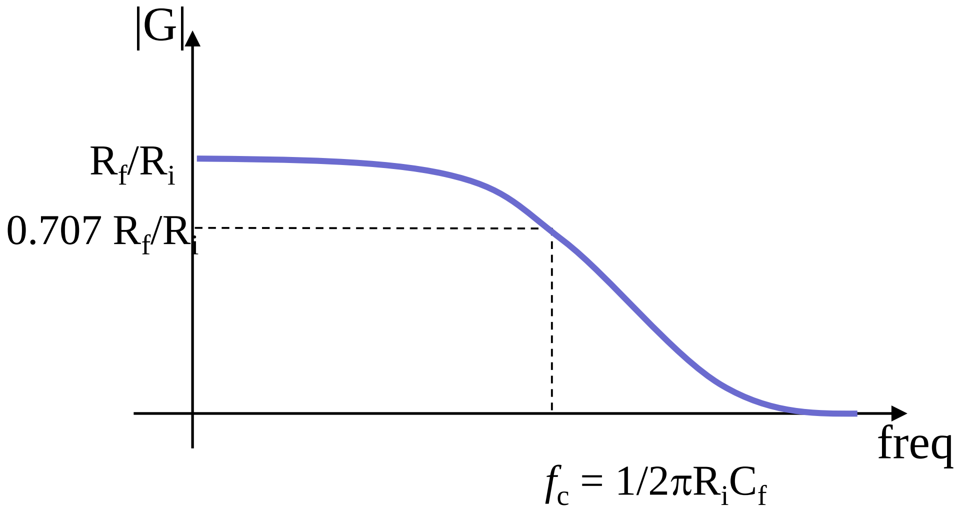


$$v_o(t) = -\frac{1}{RC}\int_0^t v_{in}(t)dt$$

Operational Amplifier (Op-amp)

- **Active Filters- Low-Pass Filter**
  - A low-pass filter attenuates high frequencies

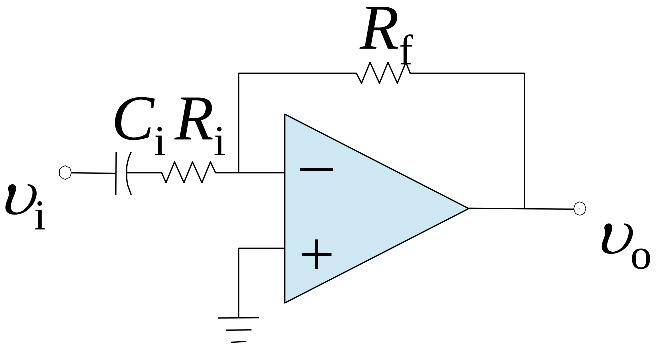


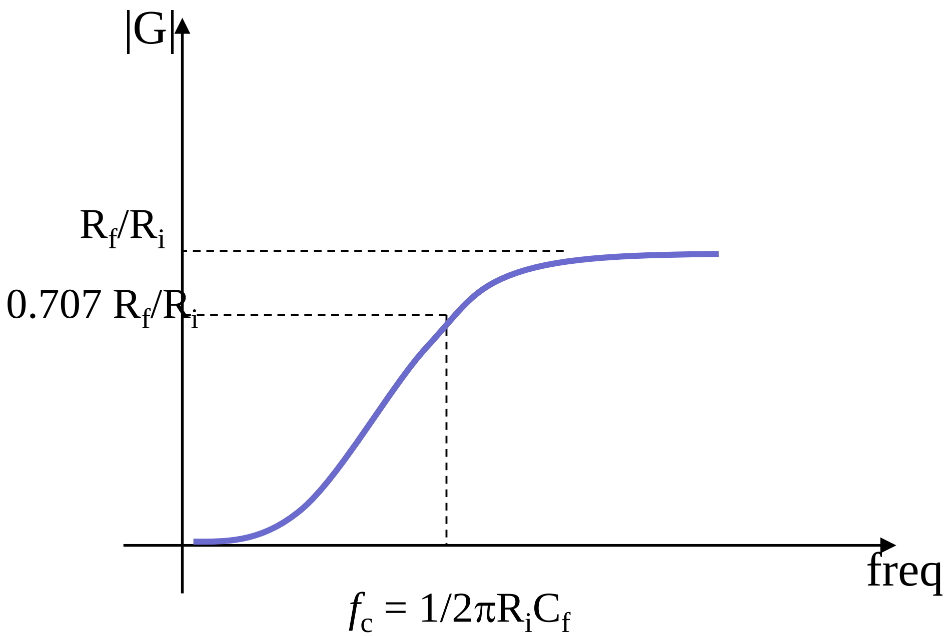


$$\text{Gain} = \text{G} = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{-R_f}{R_i}\frac{1}{1 + j\omega R_f C_f}$$

Operational Amplifier (Op-amp)

- **Active Filters (High-Pass Filter)**
  - A high-pass filter attenuates low frequencies and blocks dc.

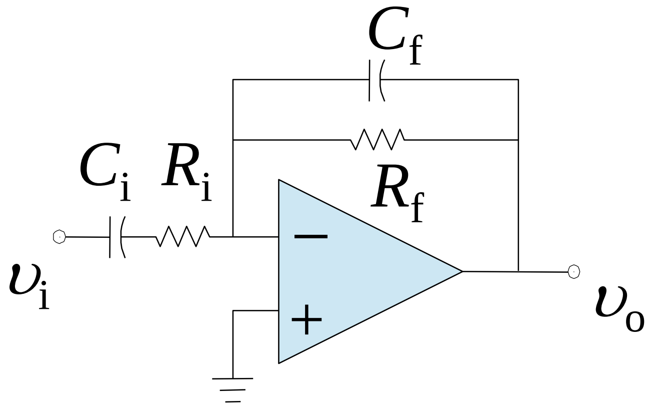




$$\text{Gain} = G = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{-R_f}{R_i}\,\frac{j\omega R_i C_i}{1 + j\omega R_i C_i}$$

Operational Amplifier (Op-amp)

- **Active Filters (Band-Pass Filter)**
  - A bandpass filter attenuates both low and high frequencies.



$$\frac{V_o(j\omega)}{V_i(j\omega)} = \frac{-j\omega R_f C_i}{(1 + j\omega R_f C_f)(1 + j\omega R_i C_i)}$$