DESIGN OF IIR DIGITAL FILTERS
Lecture 17

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IIR DIGITAL FILTERS

Advantages:
Efficient in terms of order
- Poles create narrow-band peaks efficiently
- Arbitrarily long impulse responses with few feedback
- coefficients

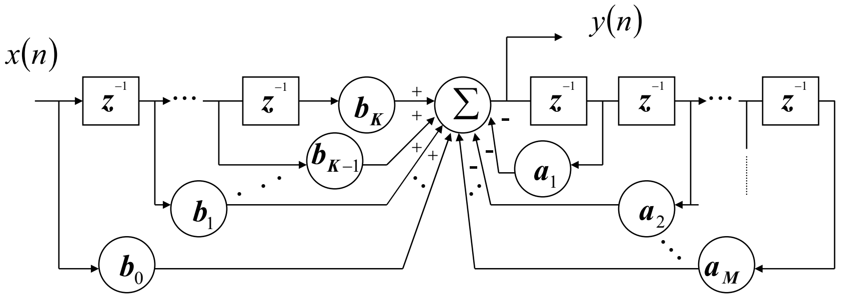
Disadvantages:
- Feedback and stability concerns
- Sensitive to Finite Word Length Effects
- Generally non-Linear Phase

Applications:
- Speech Processing, Telecommunications
- Data Processing, Noise Suppression, Radar

IIR FILTERS

The difference equation is:

$$y(n) = \sum_{i=0}^{L} b_i x(n-i) - \sum_{i=1}^{M} a_i y(n-i)$$



IIR FILTERS (Cont.)

The transfer function:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + \ldots + b_L z^{-L}}{1 + a_1 z^{-1} + \ldots + a_M z^{-M}}$$

The frequency-response function :

$$H(e^{j\Omega}) = \frac{b_0 + b_1 e^{-j\Omega} + \ldots + b_L e^{-jL\Omega}}{1 + a_1 e^{-j\Omega} + \ldots + a_M e^{-jM\Omega}}$$

IIR Filter Design by Analog Filter Approximation

The idea is to use many of the successful analog filter designs to design digital filters

This can be done by either:

- by sampling the analog impulse response (*impulse invariance*) and then determining a digital transfer function

or

- by transforming directly the analog transfer function to a digital filter transfer function using the *bilinear transformation*

IIR Filter Design by Analog Filter Approxination

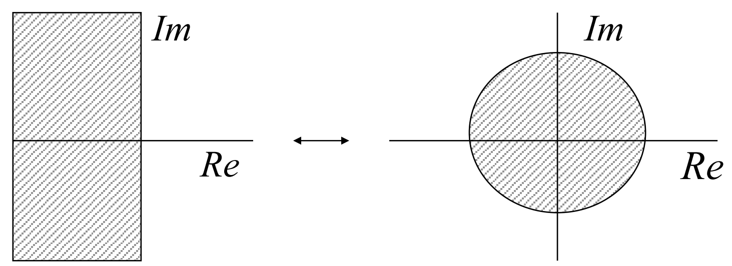
The *impulse invariance* method suffers from aliasing and is very rarely used

The *bilinear transformation* does not suffer from aliasing and is by far more popular than the impulse invariance method.
The frequency relationship from the s-plane to the z-plane is non-linear, and one needs to compensate by pre-processing the critical frequencies such that after the transformation the desired response is realized. Stability is maintained in this transformation since the left-half s-plane maps onto the interior of the unit circle.

The Bilinear Transformation

$$\textbf{\textit{Bilinear Transform}} \Rightarrow z = \frac{1+s}{1-s}$$

$S - plane$ $\qquad\qquad$ $z - plane$



$$H(z) = H\left(s = \frac{z-1}{z+1}\right)$$

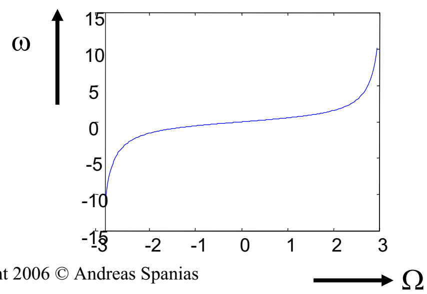
The Bilinear Transformation (Cont.)

The bilinear transformation compresses the frequency axis

$$\omega[-\infty,\infty] \leftrightarrow \Omega[-\pi,\pi]$$

The non-linear frequency transformation (frequency warping function) is given by

$$\omega = \tan\left(\frac{\Omega}{2}\right)$$



Procedure for Analog Filter Approximation

1. Consider Critical Frequencies

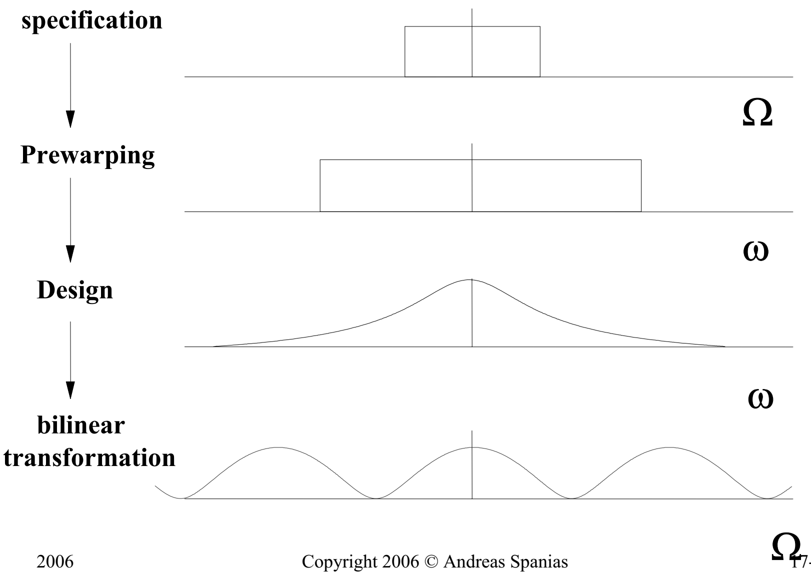
2. Pre-warp critical frequencies

3. Analog Filter Design

4. Bilinear Transformation

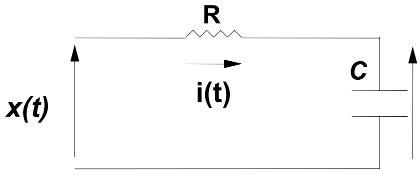
5. Realization

Applying the Bilinear Transformation



EXAMPLE: TRANSFORMING AN RC CIRCUIT TO A DF

Suppose we want a first-order (R-C LPF) appoximation



$$H(\omega) = \frac{1}{1 + j\omega RC}$$

Say we have the following DF specs:

Apply pre-warping

Step 1: $\Omega_c = \pi / 2$

Step 2: $\omega_c = \tan(\frac{\Omega_c}{2}) = \tan(\frac{\pi}{4}) = 1$

Step 3: Design the analog filter.
In this case the analog filter function is a first order LPF similar to an RC circuit →

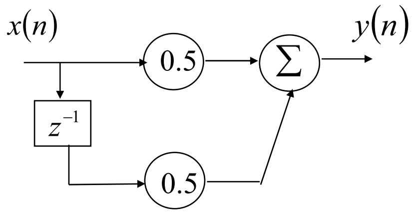
$$H(s) = \frac{1}{1+s}$$

TRANSFORMING AN RC CIRCUIT TO A DF (2)

Step 4: Apply the Bilinear Transform

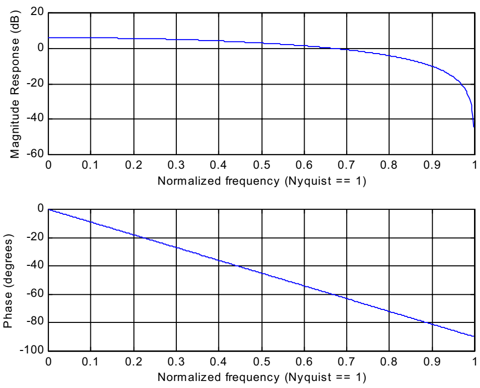
$$H(z) = H(s = \frac{z-1}{z+1}) = \frac{1}{1+\frac{z-1}{z+1}} = 0.5 + 0.5z^{-1}$$

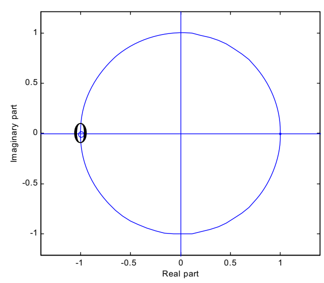
Step 5: Realization



TRANSFORMING AN RC CIRCUIT TO A DF (3)

Frequency Response $H(e^{j\Omega}) = 0.5 + 0.5e^{-j\Omega}$





Notice that there is no aliasing effect with the bilinear transformation. Although in this simple R-C example the resultant digital filter is FIR, more complex analog filters will yield IIR digital filters.