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DSP ECE 623

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 Ch6-1

MULTIRATE SIGNAL PROCESSING

QMF BANKS

Lecture 21

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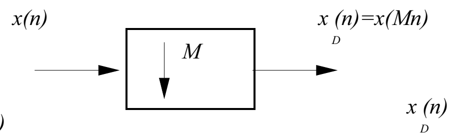
MULTIRATE SIGNAL PROCESSING AND DSP APPLICATIONS

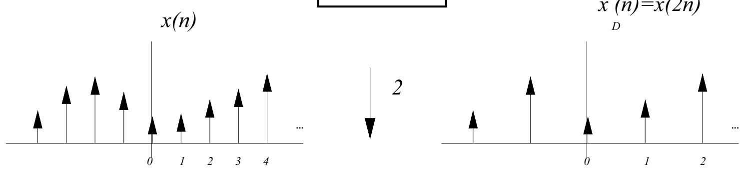
Sampling Rate Modifications are often used in many application areas.

One popular applications has been oversampling Δ-Σ A-to-D and D-to-A conversion.

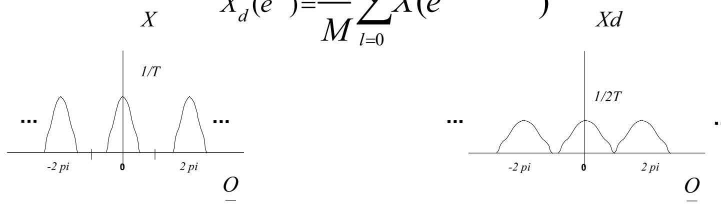
Other applications include sub-band coding of speech and audio

DOWNSAMPLING

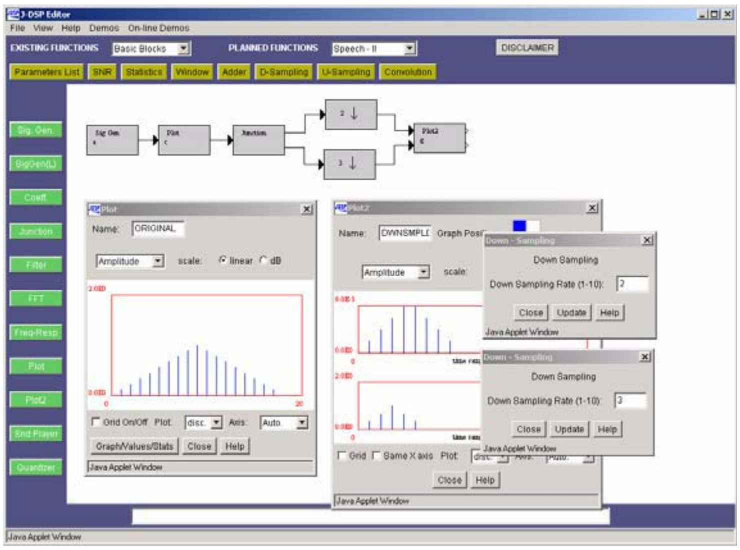




$$X_d(e^{j\Omega}) = \frac{1}{M}\sum_{l=0}^{M-1} X(e^{j(\Omega - 2\pi l)/M})$$



J-DSP and downsampling



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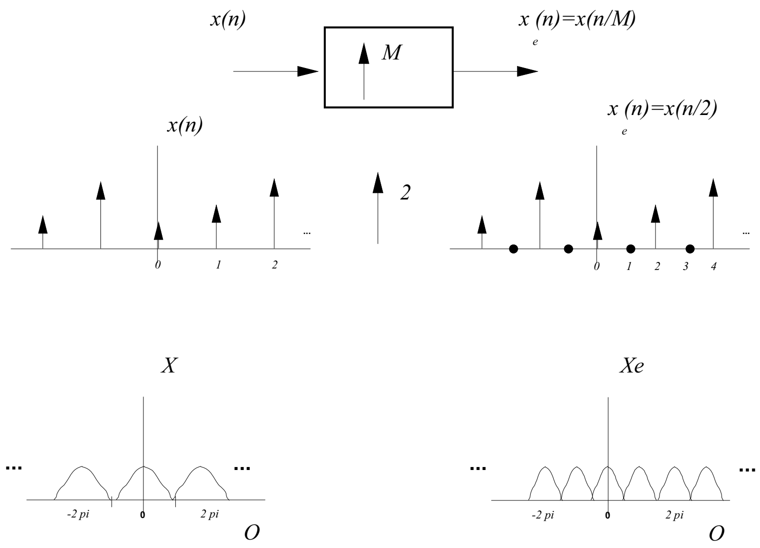
UP-SAMPLING



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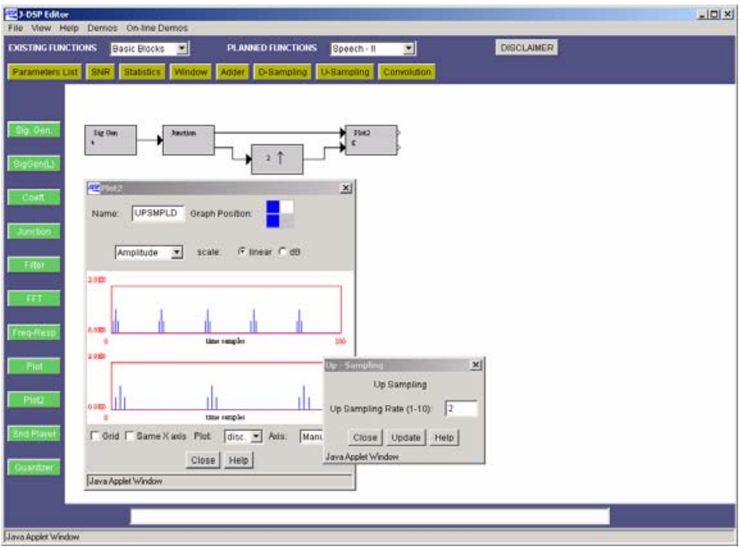
J-DSP and Upsampling



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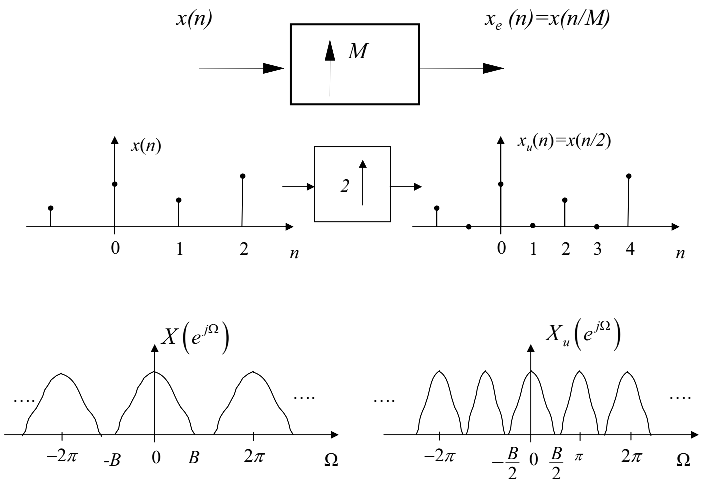
UPSAMPLING



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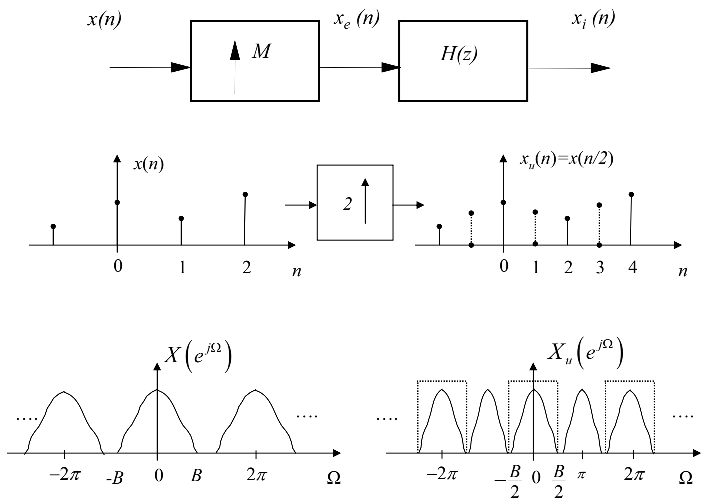
UPSAMPLING AND RECONSTRUCTION



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DOWNSAMPLING / UPSAMPLING RULES

•To avoid aliasing when downsampling a signal by an integer factor M a signal must be bandlimited by a digital antialiasing filter with cut-off frequency π/M

• If we are to upsample by a factor of M we use a digital reconstruction (interpolation) filter in order to eliminate imaging spectral components.

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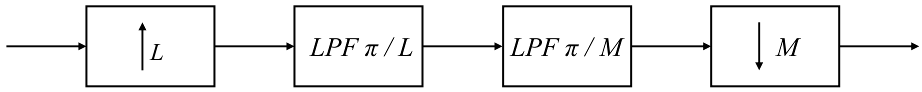
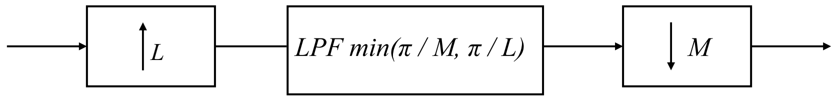
Downsampling By Non-integer Factor (L/M)



or



Example if we want to upsample by a factor 1.1
L=11 and M=10

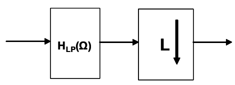
Practical Considerations in Downsampling

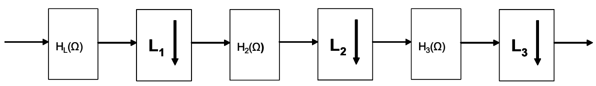
Practical choices for passband and stopband edge frequencies and tolerances defined by application requirements, i.e., bandwidth of interest and tolerable noise within that bandwidth.

Signal fidelity, not the only consideration; number of computations and memory required also considered.

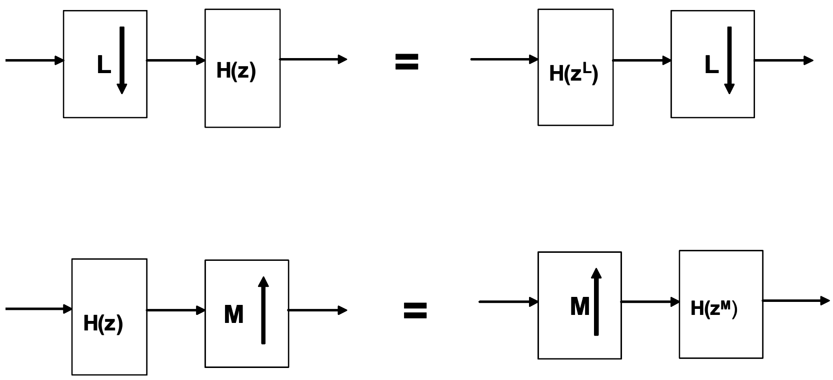
If the change in downsampling is large, the requirements on the anti-aliasing filter may require a numerically sensitive filter.

Decimation implemented gradually in multiple stages to reduce memory requirements and sensitivity.





Interchange of Operations



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DOWNSAMPLING AND A-TO-D CONVERTERS

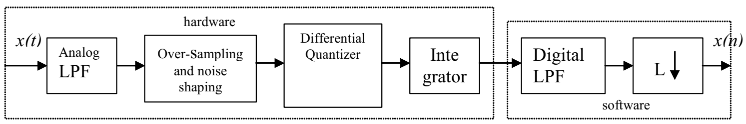
If we sample exactly at twice the signal bandwidth analog anti-aliasing filters become expensive both interms of price and chip area.

Oversampling relaxes design requirements on the analog filter thereby reducing the complexity of analog circuits.

This complexity is mapped to the digital domain where downsampling is performed in the DSP using digital antialiasing filters.

Oversampling allows use of delta modulators or difference 1 bit quantizers which are very simple to implement

The combination of oversampling and delta modulation is exploited in Δ-Σ converters



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DOWNSAMPLING AND A-TO-D CONVERTERS

If we sample at the Nyquist rate analog anti-aliasing filters become expensive both interms of price and chip area.

Oversampling relaxes design requirements on the analog filter thereby reducing the complexity of analog circuits.

This complexity is mapped to the digital domain where downsampling is performed in the DSP using digital antialiasing filters.

Oversampling allows use of delta modulators or difference 1 bit quantizers which are very simple to implement

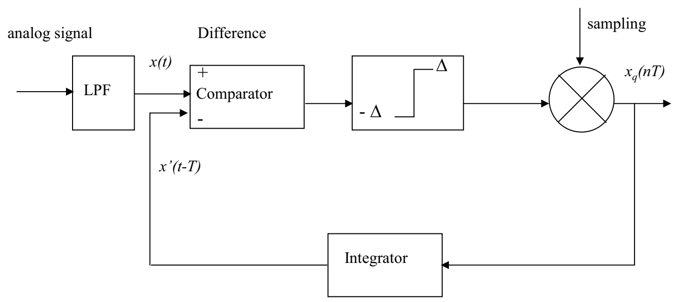
The combination of oversampling and delta modulation is exploited in Δ-Σ converters

UPSAMPLING AND D-TO-A CONVERTERS

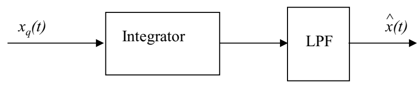
Upsampling also relaxes design requirements on the analog reconstruction filter

Upsampling is performed in the DSP using digital interpolation (reconstruction) filters.

DELTA MODULATION CIRCUITS

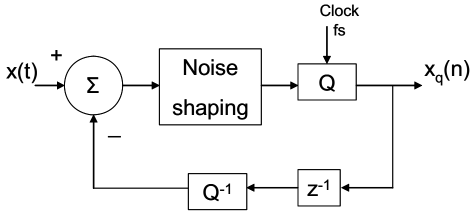


(a) Transmitter

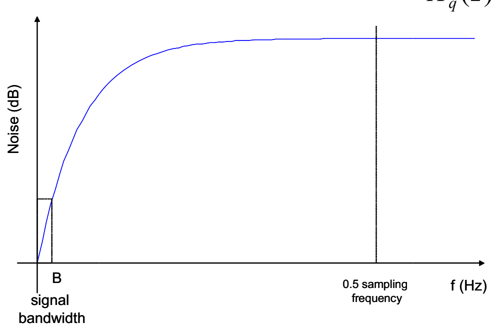


(b) Receiver

NOISE SHAPING IN DELTA -SIGMA



$$X_q(z) = X(z) + (1 - z^{-1})W(z)$$



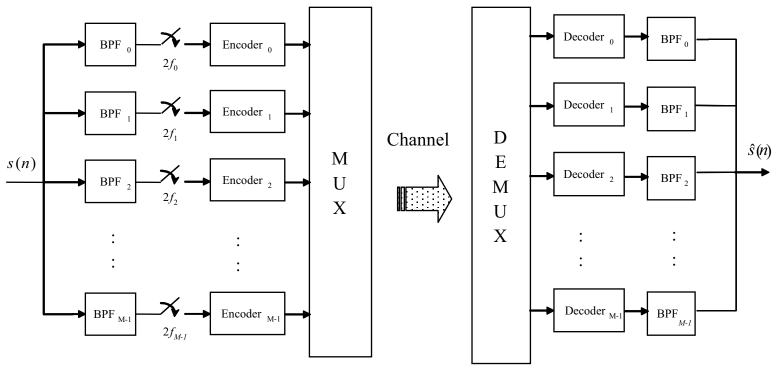
Introduction to Quadrature Mirror Filter (QMF) Banks

QMF banks can be used for signal analysis/synthesis applications

They have found applications in speech and audio coding

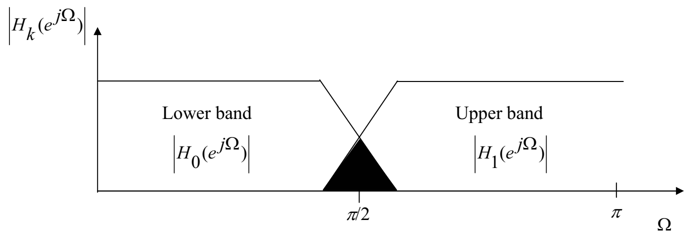
QMF involves upsampling and down sampling of a discrete signal.

Digital Filters and Sub-band Coding



- The sub-band coder (SBC) exploits the statistics of the signal and/or perceptual criteria to encode the signal in each band using a different number of bits. For example, in speech the lower frequency bands are usually allotted more bits than higher bands in order to preserve critical pitch and formant information.

Aliasing Issues



Sub-band coders not only suffer from quantization noise but also from aliasing noise due to the overlapping nature of the sub-bands. Fortunately, careful design using Quadrature Mirror Filter (QMF) banks can almost eliminate this problem.

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QMF banks

We constrain ourselves here to a simple example of a QMF for the purpose of understanding the underlying principles of aliasing cancellation. There are many other QMF design possibilities and the subject is still an open problem; with articles appearing often in the literature.

A QMF involves analysis and synthesis and incorporates:

1. Antialiasing Filters
2. Downsampling (Decimation)
3. Upsampling (Expansion)
4. Anti-imaging Filters (interpolation)

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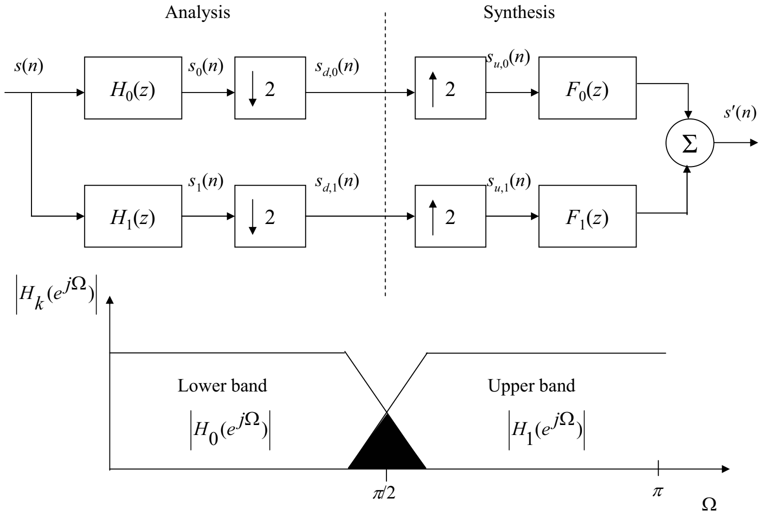
QMF banks



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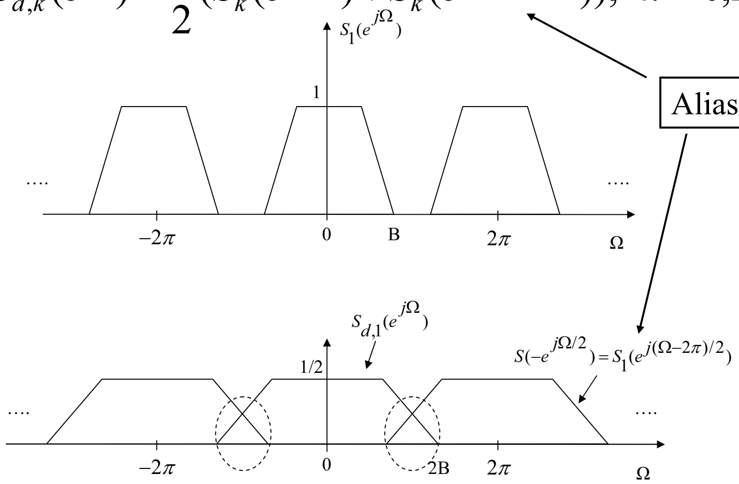
QMF banks - Aliasing Example

$$S_{d,k}(e^{j\Omega}) = \frac{1}{2}(S_k(e^{j\Omega/2}) + S_k(e^{j(\Omega-2\pi)/2})), \quad k = 0,1$$

Aliasing term



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QMF banks - Reconstruction

It can be shown that the output signal can be written as

$$S'(z) = \frac{1}{2}\left(H_0(z)F_0(z) + H_1(z)F_1(z)\right)\ \mathrm{S}(z)$$

aliasing term

$$+\frac{1}{2}\underbrace{\left(H_0(-z)F_0(z) + H_1(-z)F_1(z)\right)}_{\boxed{\text{distortion transfer function}}}\ \mathrm{S}(\text{-}\,z)$$

Remarks: It can be shown that the terms associated with (-z) are the ones responible for aliasing. For a certain choice of filters aliasing can be cancelled.

QMF banks - Reconstruction (Cont.)

Note that if we choose

$$F_0(z) = H_1(-z) \quad \text{and} \quad F_1(z) = -H_0(-z)$$

Then we have an aliased-free reconstruction. The aliased-free QMF overall transfer function is:

$$T(z) = \frac{1}{2}\left(H_0(z)F_0(z) + H_1(z)F_1(z)\right)$$

Under some conditions *T(z)* can be made all-pass and with linear phase. This is the situation where the filter bank is called a perfect reconstruction QMF.

Example of Simple QMF bank

$$H_0(z)=1+z^{-1} \quad \text{(LPF)} \qquad F_0(z)=H_1(-z)=1+z^{-1}$$

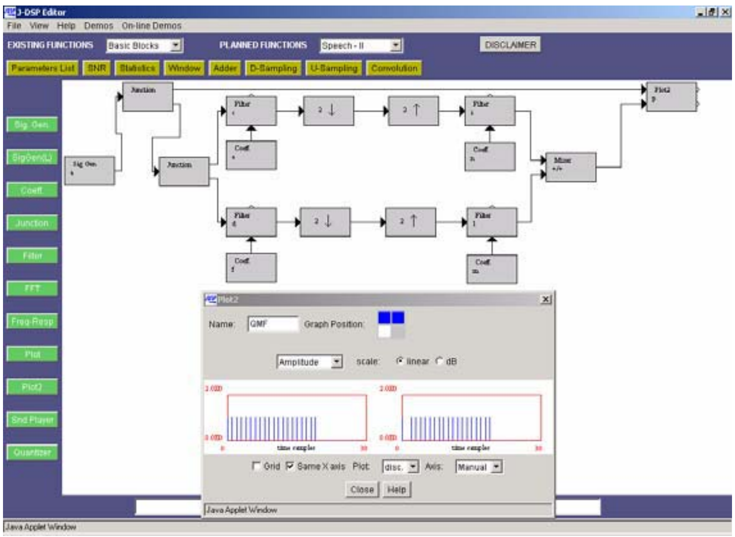
$$H_1(z)=1-z^{-1} \quad \text{(HPF)} \qquad F_1(z)=-H_0(-z)=-1+z^{-1}$$

$$T(z)=\frac{1}{2}\left((1+z^{-1})^2-(1-z^{-1})^2\right)=2z^{-1}$$

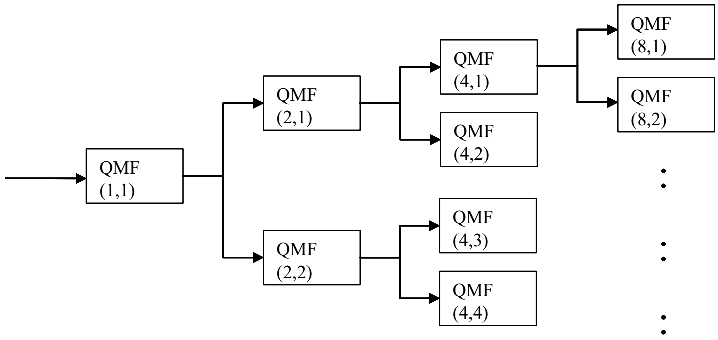
$$Distortion \;=\frac{1}{2}\left((1-z^{-1})(1+z^{-1})\;-(1+z^{-1})(1-z^{-1})\;\right)=0$$

Note that *T(z)* has only a pure delay and a gain (we can always adjust for unity the gain). Aliasing distortion is cancelled.

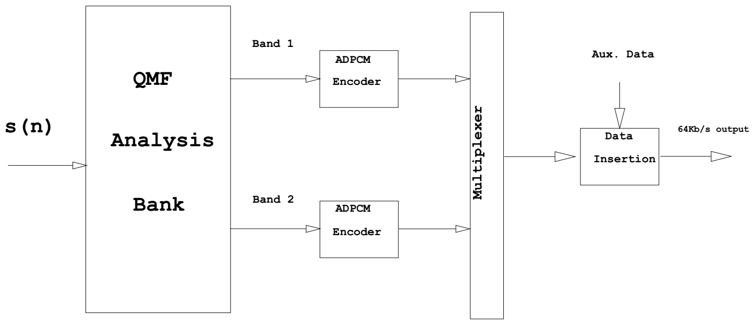
J-DSP and QMF



The Tree Structured QMF



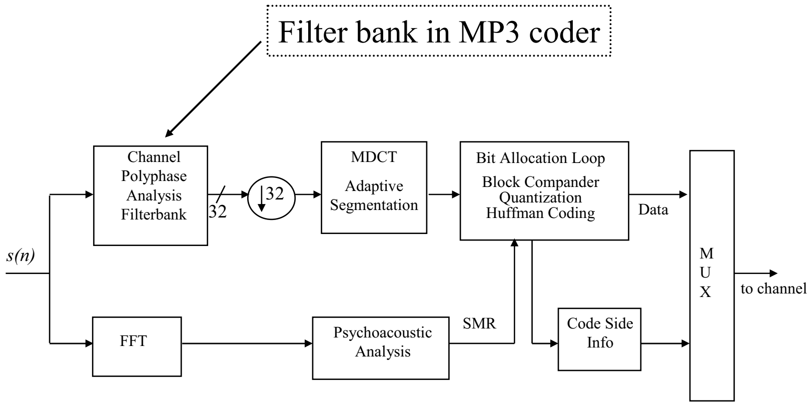
The QMF in the ITU G.722 Sub-band Coder.



- The CCITT G.722 is for wideband 7kHz audio at 64 kbits/s
- Algorithmic QMF delay less than 3 ms.

Reference: N.S. Jayant, V. Lawrence, D. Prezas, "Coding of Speech and Wideband Audio," AT&T Tech. J., Vol.69(5), pp.25-41, Sept.-Oct. 1990.

Filter bank used in the MPEG I - Layer 3 Coder (MP3)



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1. ISO/IEC-11172-3, MPEG-1 Audio Portion
2. Brandenburg, K., "ISO MPEG-1 Audio…" *J. AES*, Oct. 1994.
3. Pan, D., "A Tutorial on MPEG/Audio…", *IEEE Multimedia*, 1995.

Filterbank Responses in MP3

32 uniform subbands (750 Hz @ 48 kHz)

Magnitude Response, First 4 Channels