Digital Signal Processing

Lecture 6

Time and Frequency Response Examples

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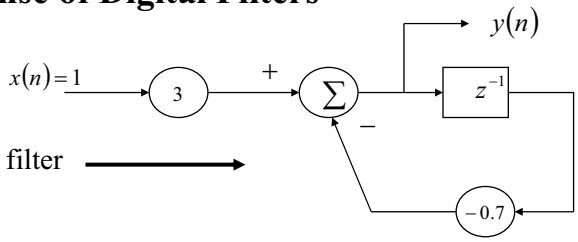
Example: Impulse, Steady-State and Transient Response of Digital Filters



Consider the first order IIR filter ⟶

$$y(n)=3x(n)+0.7y(n-1)$$

Impulse Response

$$y(n)=3x(n)+0.7y(n-1)$$

$$h(n)=3\delta(n)+0.7h(n-1)$$

Solve $h(n)-0.7h(n-1)=0$ Try $h(n)=cp^n$

$$cp^n-0.7cp^{n-1}=0$$ p=0.7

$$h(0)=3$$ c=3

Transient and Steady State Response

Same aux eq as before thus $y^{tr}(n)=c_1(0.7)^n, n\geq 0$

Since input is 1 try root 1^n $y^{ss}(n)=c_2(1)^n, n\geq 0$

$$y^{total}(n)=y^{tr}(n)+y^{ss}(n)=c_1(0.7)^n+c_2, n\geq 0$$

Initial Conditions

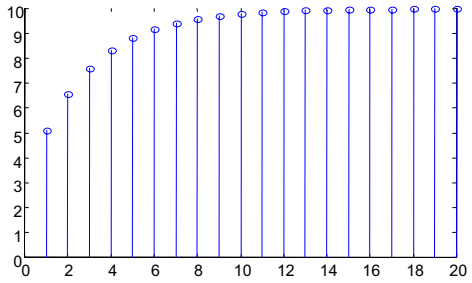
$$y^{total}(0)=3 \qquad y^{total}(1)=5.1$$

Steady-State and Transient Response (2)

$$c_1 + c_2 = 3 \qquad c_1 = -7$$

$$0.7c_1 + c_2 = 5.1 \qquad c_2 = 10$$

$$y(n) = 10 - 7(0.7)^n, n \geq 0$$



Example: Find the Impulse Response of the Filter

$$y(n) = x(n) + 5/6\,y(n-1) - 1/6\,y(n-2)$$

$$h(n) - 5/6\,h(n-1) + 1/6\,h(n-2) = 0$$

$$p^n - 5/6\,p^{n-1} + 1/6\,p^{n-2} = 0$$

$$p^2 - 5/6\,p + 1/6 = 0$$

$$p_1 = 1/2$$

$$p_2 = 1/3$$

Example: Find the Impulse Response of the Filter

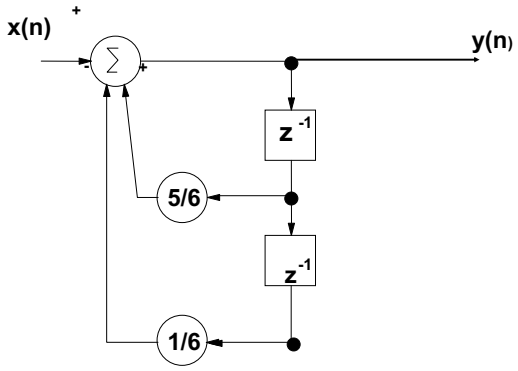
$$h(n) = c_1\ (1/2)^n + c_2\ (1/3)^n \quad n \geq 0$$

$$h(0) = c_1 + c_2 = 1$$

$$h(1) = c_1/2 + c_2/3 = 5/6$$

$$h(n) = 3\left(\frac{1}{2}\right)^n - 2\left(\frac{1}{3}\right)^n, \; n \geq 0$$

Structure



$$y(n) = x(n) + 5/6\,y(n-1) - 1/6\,y(n-2)$$

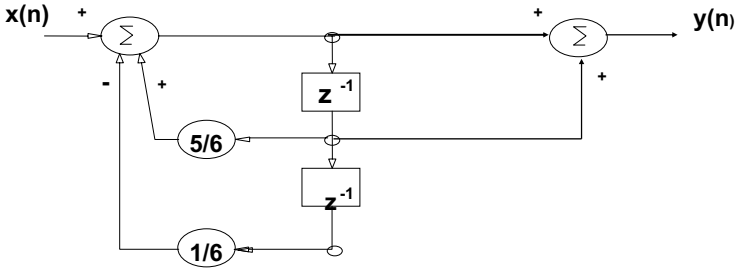
Structure (2)

Consider the modified filter below (add a unit delay on input)

$$y(n) = x(n) + x(n-1) + 5/6\,y(n-1) - 1/6\,y(n-2)$$

Then its impulse response is:

$$h(n) = \left(3\left(\frac{1}{2}\right)^n - 2\left(\frac{1}{3}\right)^n\right)u(n) + \left(3\left(\frac{1}{2}\right)^{n-1} - 2\left(\frac{1}{3}\right)^{n-1}\right)u(n-1)$$



Interesting Filters and Frequency Responses

$$y(n) = x(n) - a_p y(n-p)$$

•Used in Vocoders

Often called Long-term Predictor Synthesis Filter
Tends to create periodic output
Example p=10>>

$$y(n) = x(n) + 0.9y(n-10)$$

Interesting Frequency Responses

$$H(z) = \frac{1}{1 - 0.9e^{-j\Omega 10}}$$