ECE 631 Homework # 1

(due on Monday, 1 February 2016 at 3:00pm)

- **Reading assignment:** Read Class Notes and read from book *Fundamentals of Linear State Space Systems* by John Bay, Chapter 1 - Models of Linear Systems and Chapter 2 - Vectors and Vector Spaces.

1. (20 pts)

   (a) Let Y_1, Y_2 be subspaces of a linear space Y. Show that $Y_1 \bigcup Y_2$ is not necessarily a subspace of Y.

   (b) Show that for any linear operator $\mathcal{A} : X \mapsto Y$, the following holds:

   $$\mathcal{A} \ \textit{is injective} \iff \mathcal{N}(\mathcal{A}) = \{0_X\}$$

   (The symbol $\iff$ means "if and only if." In other words Statement A $\iff$ Statement B means that "Statement A implies Statement B" and also "Statement B implies Statement A." The symbol $\mathcal{N}(\mathcal{A})$ denotes the null space of $\mathcal{A}$).

2. (20 pts) Consider the linear vector space $(\mathcal{R}^3, \mathcal{R})$. Let

   $$\begin{aligned} X_1 &= \left\{ x \in \mathcal{R}^3 \ \mid \ x_1 - 2x_2 + 3x_3 = 0 \right\} \\ X_2 &= \left\{ x \in \mathcal{R}^3 \ \mid \ 2x_1 - x_1 x_2 - x_3 = 0 \right\} \\ X_3 &= \left\{ x \in \mathcal{R}^3 \ \mid \ x_1^3 - x_2 = 1 \right\} \\ X_4 &= \left\{ x \in \mathcal{R}^3 \ \mid \ |x_1 + x_3| \leq 1 \right\} \end{aligned}$$

   Which of the above are subspaces of $(\mathcal{R}^3, \mathcal{R})$? (give a formal proof)

3. (10 pts) The set $C^3[0, \infty)$ denotes the class of functions with (at least) three continuous derivatives defined on the interval $0 \leq t < \infty$. It is easy to see that $(C^3[0, \infty),\ \mathcal{R})$ is a linear space of infinite dimension. Show that

   $$X = \left\{ x \in C^3[0, \infty) \ \mid \ \frac{d^3x}{dt^3} + t\frac{d^2x}{dt^2} - t^2\frac{dx}{dt} - x = 0 \right\}$$

   (i.e., the class of functions satisfying the above differential equation) is a linear subspace of $(C^3[0, \infty),\ \mathcal{R})$.

4. (20 pts)

   (a) Consider the linear space $(\mathcal{P}_3(x),\ \mathcal{R})$, where $\mathcal{P}_3(x)$ denotes the class of polynomials of (at most) degree 3, defined on the interval $0 \le x \le 1$. Is the set

$$\left\{x^3 - 4x^2 - 7x - 9, \quad 2x^3 - 2x^2 - x + 1, \quad x^2 - 3x - 1\right\}$$

   linearly independent in $(\mathcal{P}_3(x),\ \mathcal{R})$? Justify your answer.

   (b) Consider the linear space $(\mathcal{P}_2(x),\ \mathcal{R})$, where $\mathcal{P}_2(x)$ denotes the class of polynomials of degree 2, defined on the interval $-\infty < x < \infty$. Is the set

$$\left\{2x^2 + 7, \quad -x^2 + x - 1, \quad 4x + 10\right\}$$

   linearly independent? Justify your answer.

   (c) Consider the linear space $([\mathcal{P}_2(x)]^{2\times 2},\ \mathcal{R})$, where $\mathcal{P}_2(x)$ is as defined in (b). Is the set

$$\left\{\begin{bmatrix} x^2 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} x^2 & x^3 \\ 0 & 0 \end{bmatrix}\right\}$$

   linearly independent? Justify your answer.

5. (10 pts) (**Introduction to MATLAB**) This exercise is intended to familiarize you with the MATLAB software package (for those already familiar with MATLAB, it will be a simple review). Get a computer account (if you don't already have one) and enter matlab by typing "matlab". Type

   >> intro

   and follow the instructions of the introduction to MATLAB. Then type

   >> demo

   and play with the demonstration routines. Submit a labelled plot of the function

$$y(x) = \sin(\pi x) + \tanh(2x) + \frac{2.3x^2}{2 + \sin(4\pi x)} \qquad -1 \le x \le 2$$

   Try "help help", "help plot", "help title", "help xlabel", "help ylabel", and "help print".