ECE 631 Homework # 2

(due on Monday, 22 February 2016 at 3:00pm)

- **(Reading assignment)** Read Class Notes and read from book *Fundamentals of Linear State Space Systems* by John Bay, Chapters 3 and 4.

1. Let the linear operator $\mathcal{A} : \mathcal{R}^3 \mapsto \mathcal{R}^4$ be defined as

$$\mathcal{A}(x) = \mathcal{A}(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}) = \begin{bmatrix} 2x_1 - 3x_3 \\ -x_1 + x_2 - 2x_3 \\ -x_1 + x_3 \\ -4x_3 \end{bmatrix}.$$

Consider the following bases for $\mathcal{R}^3$ and $\mathcal{R}^4$:

$$u = \left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \right\} \qquad \tilde{u} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix} \right\}$$

$$v = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \right\} \qquad \tilde{v} = \left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \right\}$$

Find $A = [\mathcal{A}]_{v,u}$ and $\tilde{A} = [\mathcal{A}]_{\tilde{v},\tilde{u}}$. Verify that $A = Q\tilde{A}P^{-1}$.

2. (a) Let $\mathcal{A} : \mathcal{R}^4 \mapsto \mathcal{R}^3$ be defined as

$$\mathcal{A}(x) = \mathcal{A}(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}) = \begin{bmatrix} 0 \\ x_1 - x_4 \\ 0 \end{bmatrix}.$$

   (i) Show that $\mathcal{A}$ is a *linear* operator.

   (ii) Find a set of basis vectors for the null space $\mathcal{N}(\mathcal{A})$.

   (iii) Find a set of basis vectors for the range space $\mathcal{R}(\mathcal{A})$.

   (b) Repeat (a) (parts (ii) and (iii) for the operator $\mathcal{A}_0 : \mathcal{R}^3 \mapsto \mathcal{R}^4$ where

$$\mathcal{A}_0(x) = \mathcal{A}_0(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}) = \begin{bmatrix} 2x_1 + x_3 \\ x_2 - x_3 \\ 4x_1 + 2x_2 \\ 0 \end{bmatrix}.$$

3. (a) Compute the eigenvalues, eigenvectors, and Jordan form for the matrix

$$A = \begin{bmatrix} -2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}.$$

   and verify that $\mathrm{Trace}(A) = \sum_{i=1}^{3} \lambda_i$, where $\lambda_1, \lambda_2, \lambda_3$ are the eigenvalues of A.

(b) If two matrices A and B are related by a change of basis (that is, $B = P^{-1}AP$ for some P), then the matrices are said to be *similar*. Prove that similar matrices have the same characteristic polynomial.

4. Check whether the function $f : \mathcal{R}^2 \mapsto \mathcal{R}$ given by

$$f(x) = f\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \left(x_1^2 + 2x_1x_2 + 4x_2^2\right)^{1/2}$$

defines a norm on $\mathcal{R}^2$.

5. Compute $\|A\|_1$, $\|A\|_2$, $\|A\|_\infty$ for the matrices

(a) $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, (b) $\begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}$, (c) $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, (d) $\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$

6. Consider the linear space $C[0,\ 5]$ (that is, the set of continuous functions defined on the interval $0 \le t \le 5$). Compute $\|x\|_1$, $\|x\|_2$, $\|x\|_\infty$ for the functions

(a) $e^{t/3} - t$ (you may use MATLAB to compute any zero-crossings).

(b) $\sin(kt)$ (the answer will be a function of k).

7. Find the point on the line $3x + 2y = 5$ in two dimensional space closest to the origin when distance is measured by each of the following three norms: (a) the 1-norm, (b) the 2-norm, (c) the ∞-norm.

8. (**MATLAB assignment**) This exercise is intended to familiarize you with some matrix manipulation functions of MATLAB.

Using File − > New − > M-File, create a file in the same directory as your MATLAB environment and name it "filehw2.m" ; this is called an M-File. Write the following in "filehw2.m":

```
for i = 1:7
for j = 1:7
A(i, j) = (4*rand)-2 ;
end
end
A
```

This will create a matrix of dimension 7 by 7 with random entries between -2 and 2. Now, type
>> filehw2
Calculate the following of your matrix A: (a) determinant; (b) eigenvalues, eigenvectors ; (c)

condition number; (d) rank; (e) inverse; (f) trace; (g) characteristic polynomial; (h) norm; (i) null space; (j) singular value decomposition.

Create a “diary” of your MATLAB session and submit it with your homework. (try “help diary”).

Do “help” on the following MATLAB commands to obtain on-line information on how to calculate the above quantities: “det”, “eig”, “cond”, “rank”, “inv”, “trace”, “poly”, “norm”, “null”, “svd”. (do *not* include these “help” commands in the diary that you turn in.)